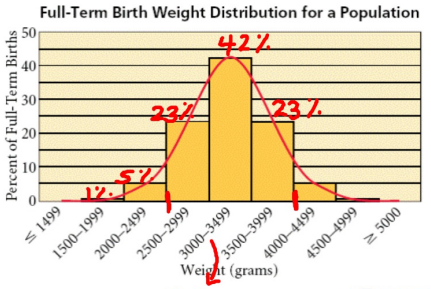
12-7 Normal Distributions

Normal Distribution – Shows data that vary randomly from the mean.

Normal Curve – A bell-shaped curve the pattern of the data forms.

Example 1: The bar graph below gives the birth weights of a population of 100 babies. The red curve shows how the weights are normally distributed about the mean, 3250 g.



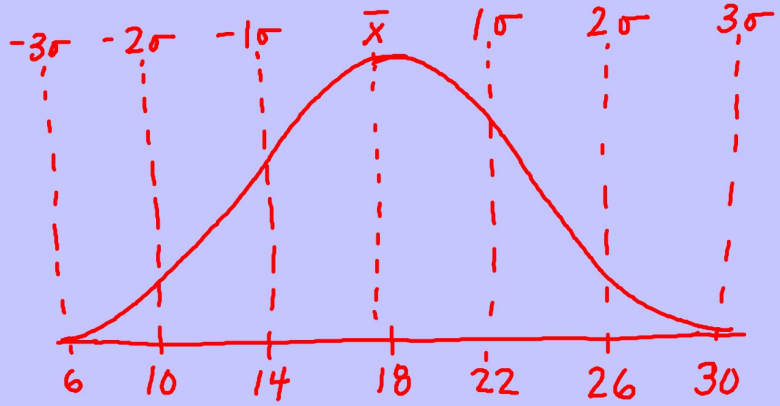
3250 − 750 = 2500 3250 + 750 = 4000

A. Estimate the percent of babies weighing less than 3500 g. 71%.

B. The standard deviation in birth weights is about 500 g. Estimate the percent of babies whose birth weights are within 1.5 standard deviations of the mean. 1.5(500) 750

23+42+23 = 88%

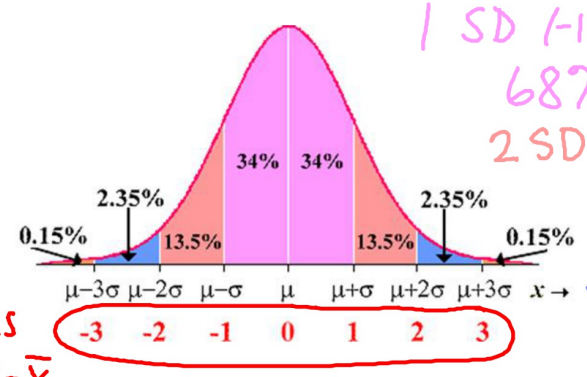
Example 2: A survey of the employees of the XYZ Corporation found that the mean morning commute time to work was 18 minutes. The standard deviation was 4 minutes. Sketch a normal curve showing the commute times at one, two, and three standards deviations from the mean.



When you show a probability distribution as a bar graph, the height of the bar for each outcome indicates the probability. For a normal curve, however, the area between the curve and the x-axis represents the probability.

Standard Normal Curve – A normal distribution centered on the ~~x~~-axis. The mean is 0 and the standard deviation is 1.

1.



1 SD (-1σ to 1σ) 68%

2 SD (-2σ to 2σ) 95%

3 SD 99.7%

Z-Scores $z = \frac{x - \bar{x}}{\sigma}$

When a data set is normally distributed, abut 68% of the data fall within one standard deviation of the mean and 95% fall within two standard deviations of the mean.

To find the values that are two standard deviations away from the mean, find the values that have z-scores of –2 and 2.

Example 3: In a survey, the responses to the questions, "How much time do you spend in the shower every day?" were normally distributed. The mean was 15 minutes; the standard deviation was 2 minutes.

A. What values are one standard deviation from the mean?

$-1 = \frac{x - \bar{x}}{\sigma}$ $-1 = \frac{x-15}{2}$ $-2 = x - 15$ $13 = x$

-1σ 1σ 13m – 17m

$1 = \frac{x-15}{2}$ $2 = x - 15$ $x = 17$

B. What percent of responses would you expect to find from 13 min to 17 min?

68%

C. Suppose there were 200 responses to the survey question. How many responses would you expect to be values from 13 min to 17 min?

68% of 200

.68 · 200

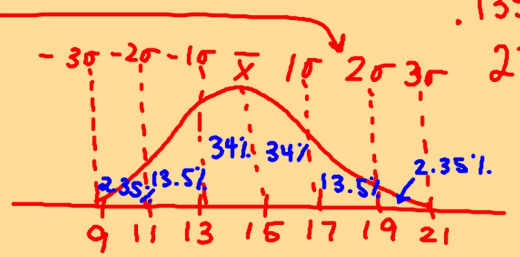
136 people

D. Of the 200 responses, how many would you expect to be values from 17 min to 19 min?

13.5%

.135 · 200

27 people



Example 4: In a university lecture class with 140 students, the final exam scores have a mean of 68.5 and a standard deviation of 7.3. The grades on the exam are all whole numbers, and the grade pattern follows a normal curve.

A. Find the number of students who receive grades from one to two standard deviations above the mean.

13.5% of 140

.135 · 140 = 18.9 ↑ 19 stus

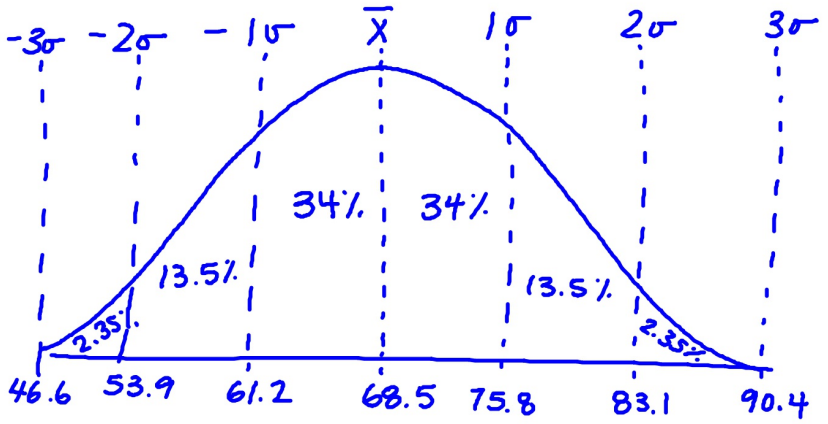
B. Find the number of students who receive a grade of 61 or below.

≈ 16% of 140

.16 · 140 = 22.4 ↓ 22 students

C. Find the number of students who receive grades from 69 to 75.

.34 · 140 = 47.6 ↑ 48 students



H.W. p. 681-682 # 2-24 even