1-5 Absolute Value equations and inequalities

The absolute value of a number is its distance from zero on a number line. Since absolute value represents distance, it can never be negative.

How do you know if the expression in the absolute value is positive or negative?
ex: $|x| = 4$

An absolute value equation has two solutions.

Solve each equation. Check your work.

$|3x+2| = 7$

L

$3x + 2 = -7$
$\quad -2 \quad -2$
$3x = -9$
$\div 3 \quad \div 3$
$x = -3$

$|3(-3)+2|$
$|-9+2|$
$|-7| \stackrel{\checkmark}{=} 7$

R

$3x + 2 = 7$
$\quad -2 \quad -2$
$3x = 5$
$\div 3 \quad \div 3$
$x = \frac{5}{3}$

$|3(\frac{5}{3})+2|$
$|5+2|$
$7 \stackrel{\checkmark}{=} 7$

Solve each equation. Check your work.

$|15 - 3x| = 6$

L

$15 - 3x = -6$
$-15 \quad -15$
$\frac{-3x}{-3} = \frac{-21}{-3}$
$x = 7$

$|15 - 3(7)|$
$|15 - 21|$
$|-6| \stackrel{\checkmark}{=} 6$

R

$15 - 3x = 6$
$-15 \quad -15$
$\frac{-3x}{-3} = \frac{-9}{-3}$
$x = 3$

$|15 - 3(3)|$
$|15 - 9|$
$6 \stackrel{\checkmark}{=} 6$

You must first isolate the absolute value expression on one side of the equation before solving.
Solve each equation. Check your work.

$2|3x - 1| + 5 = 33$
$\quad -5 \quad -5$
$\frac{2|3x-1|}{2} = \frac{28}{2}$
$|3x - 1| = 14$

L

$3x - 1 = -14$
$\frac{3x}{3} = \frac{-13}{3}$
$x = -\frac{13}{3}$

$2|3(-\frac{13}{3}) - 1| + 5 = 33$
$2|-14| + 5 = 33$
$2(14) + 5 = 33$
$33 \stackrel{\checkmark}{=} 33$

R

$3x - 1 = 14$
$3x = 15$
$x = 5$

$2|3(5) - 1| + 5 = 33$
$2|15 - 1| + 5 = 33$
$2(14) + 5 = 33$
$33 \stackrel{\checkmark}{=} 33$

Solve each equation. Check your work.

$4-2|x+9| = -5$

$-4 \qquad\quad -4$

$\frac{-2|x+9|}{-2} = \frac{-9}{-2}$

$|x+9| = 4.5$

L

$x+9 = -4.5$

$-9 \quad -9$

$x = -13.5$

$4-2|-13.5+9| = -5$

$4-2|-4.5| = -5$

$4-2(4.5) = -5$

$4-9 = -5$

$-5 \overset{\checkmark}{=} -5$

R

$x+9 = 4.5$

$-9 \quad -9$

$x = -4.5$

$4-2|-4.5+9| = -5$

$4-2(4.5) = -5$

$4-9 = -5$

$-5 \overset{\checkmark}{=} -5$

An extraneous solution is a solution of an equation that is derived from an original equation but is not a solution of the original equation.

Solve each equation. Check your work.

$|2x+3| = 3x+2$

L

$2x+3 = -3x-2$

$+3x \qquad +3x$

$5x+3 = -2$

$5x = -5$

$x \neq -1$ extraneous solution

$|2(-1)+3| = 3(-1)+2$

$|-2+3| = -3+2$

$1 \neq -1$

R

$2x+3 = 3x+2$

$-x+3 = 2$

$-x = -1$

$x = 1$

$|2(1)+3| = 3(1)+2$

$|2+3| = 3+2$

$5 \overset{\checkmark}{=} 5$

Solve each equation. Check your work.

$|3x-4| = -4x-1$

L

$3x-4 = 4x+1$

$-5 = x$

$|3(-5)-4| = -4(-5)-1$

$|-15-4| = 20-1$

$|-19| = 19$

$19 \overset{\checkmark}{=} 19$

$\frac{4 \cdot 7}{1 \cdot 7} \quad \frac{28}{7}$

R

$3x-4 = -4x-1$

$7x = 3$

$x \neq \frac{3}{7}$ extraneous solution

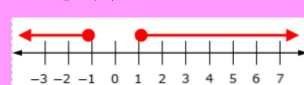
$\left|3\left(\frac{3}{7}\right)-4\right| = -4\left(\frac{3}{7}\right)-1$

$\left|\frac{9}{7}-4\right| = -\frac{12}{7}-1$

$\left|\frac{9}{7}-\frac{28}{7}\right| = -\frac{12}{7}-\frac{7}{7}$

$\frac{19}{7} \neq -\frac{19}{7}$

Graph $|x| \geq 1$

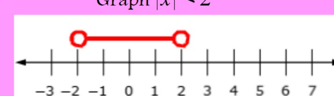


$(-\infty, -1]$ $\quad$ $[1, \infty)$

$1 \leq x < \infty$

$-\infty < x \leq -1$ (Remember: GOAL: > or, and <)

Graph $|x| < 2$



$(-2, 2)$

$-2 < x < 2$

Interval notation is helpful because it shows the same information as the graph more consisely, using only two numbers. $\geq$ and $\leq$ use [ ] square brackets $<$ and $>$ use ( ) round brackets. Write the smallest number in the interval first, and the largest number in the interval second.

Let k represent a positive real number.

$|x| \geq k$ is equivalent to $x \leq -k$ or $x \geq k$ (or)

$|x| \leq k$ is equivalent to $-k \leq x \leq k$ (and)

AND-conjunction-both true-INTERSECTION

OR-disjunction-either true-UNION

When an absolute value is combined with other operations, first isolate the absolute value expression on one side of the inequality.

Don't forget, when multiplying or dividing by a negative number, YOU MUST FLIP THE SIGN!

Solve each inequality. Then graph and write the answer in interval notation.

$|2x-3| > 7$ ← or →

L

$2x-3 < -7$

$2x < -4$

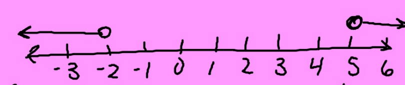
$x < -2$

R

$2x-3 > 7$

$2x > 10$

$x > 5$



$(-\infty,-2)$ or $(5,\infty)$

Solve each inequality. Then graph and write the answer in interval notation.

$|2x-5| \leq 3$ and

L

$2x-5 \geq -3$

$2x \geq 2$

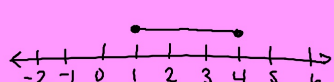
$x \geq 1$

R

$2x-5 \leq 3$

$2x \leq 8$

$x \leq 4$



$[1,4]$

Solve each inequality. Then graph and write the answer in interval notation.

$|5x+3|-7<34$

Solve each inequality. Then graph and write the answer in interval notation.

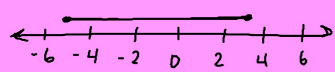
$-2|x+1|+5 \geq -3$

$-5 \quad -5$

$\dfrac{-2|x+1|}{-2} \geq \dfrac{-8}{-2}$

$|x+1| \leq 4$

$[-5, 3]$



and

L

$x+1 \geq -4$

$x \geq -5$

R

$x+1 \leq 4$

$x \leq 3$

You can use absolute value inequalitites and compound inequalities to specify allowable ranges in measurements.

Tolerance is the difference between a desired measurement and its maximum and minimum values.

Tolerance equals one-half of the difference between the maximum and minimum values.

The specifications for the circumference, C, in inches of a men's basketball is

$$29.5 \leq C \leq 30$$

Write the specification as an absolute value inequality.

tolerance $\begin{array}{r} 30 \\ -29.5 \\ \hline .5 \end{array}$ $\dfrac{.5}{2} = .25$

perfect bball $\dfrac{29.5+30}{2}$ $= 29.75$

$|C - 29.75| \leq 0.25$

Homework:

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