10.1 Exploring Conic Sections

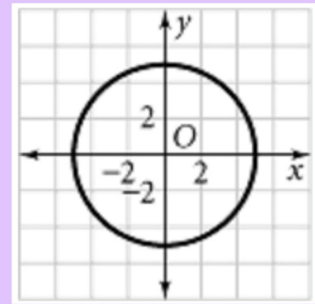
10.2a Parabolas

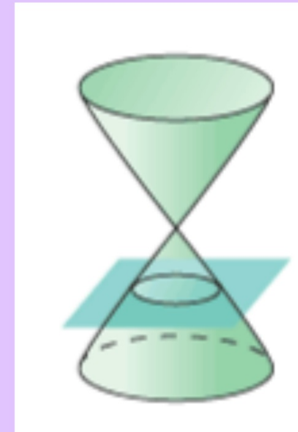
CONIC SECTION – The shape created by the intersection of a plane and a double cone.

By changing the inclination of the plane, you can create a circle, a parabola, an ellipse, or a hyperbola.

CIRCLE: created by a horizontal plane

Example equation: $x^2 + y^2 = 25$



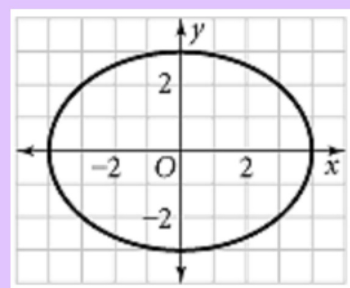


PARABOLA: Created by an oblique plane passing through the base.

Example equation: $y = 2x^2$

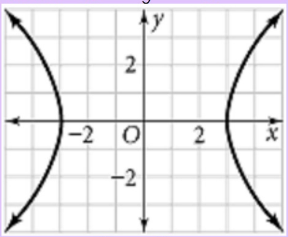
ELLIPSE: created by an oblique plane

Example equation: $9x^2 + 16y^2 = 144$

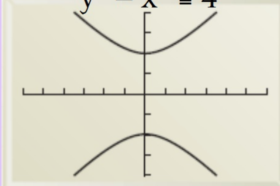


HYPERBOLA: created by a vertical plane.

Example equation: $x^2 - y^2 = 9$



$y^2 - x^2 = 4$



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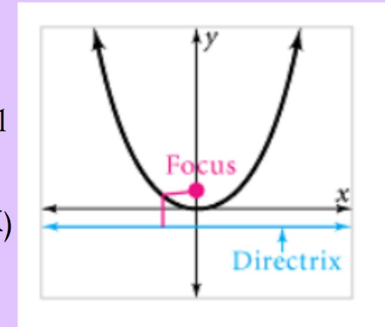
A conic section we have previously worked with is the parabola.

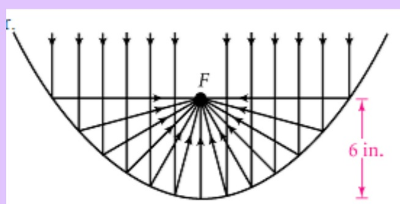
$y = a(x - h)^2 + k$ Vertex: (h, k)

Axis of symmetry: $x = h$

The actual definition of a parabola is… All the points in a plane that are the same distance from a fixed line (the **DIRECTRIX**) and a fixed point (the **FOCUS**) not on the line. The line through the focus and perpendicular to the directrix is the axis of symmetry.



Parabolas play an important role in many applications. Headlights, Solar Mirrors, Radio dishes. In physics, you will learn that light beams which approach a parabolic surface parallel to the axis of symmetry are reflected to the focus of the parabola. These facts are used in the construction of objects such as solar collectors and flashlights.



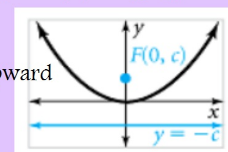
Consider any equation $y = x^2$ (opening up/down)

If $a > 0$, then

*the parabola opens upward

*the focus is at $(0, c)$

*the directrix is $y = -c$

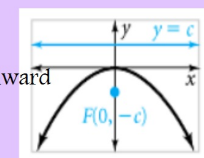


If $a < 0$, then

*the parabola opens downward

*the focus is at $(0, -c)$

*the directrix is $y = c$



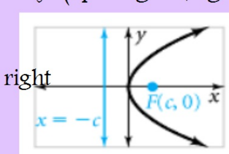
Consider any equation $x = y^2$ (opening left/right)

If $a > 0$, then

*the parabola opens to right

*the focus is at $(c, 0)$

*the directrix is $x = -c$

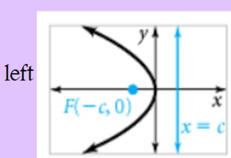


If $a < 0$, then

*the parabola opens to left

*the focus is at $(-c, 0)$

*the directrix is $x = c$



To symbolize the distance between the vertex and the focus, we use the letter *c*.

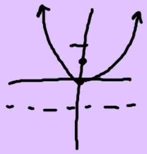
To make a connection between the equation and the focus (*c*), we use the following: $|a| = \frac{1}{4c}$

Write an equation for a graph that is the set of all points in the plane that are equidistant from focus and directrix.

F (0,1) directrix: y = –1

$|a| = \frac{1}{4(1)} = \frac{1}{4}$

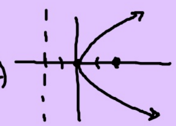
$y = \frac{1}{4}x^2$



F (2, 0) directrix: x = –2

$|a| = \frac{1}{4(2)}$

$|a| = \frac{1}{8}$



$x = \frac{1}{8}y^2$

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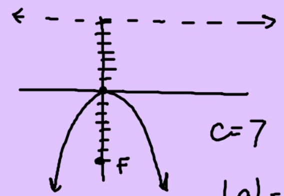
Write an equation of a parabola with a vertex at the origin and the given focus.

F (0, –7)



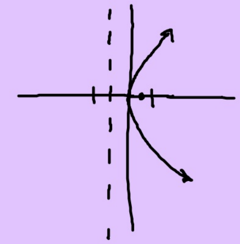
$c = 7$

$|a| = \frac{1}{4(7)}$

$|a| = \frac{1}{28}$

$y = -\frac{1}{28}x^2$

F ($\frac{1}{2}$, 0)



$x = \frac{1}{2}y^2$

$c = \frac{1}{2}$

$|a| = \frac{1}{4(\frac{1}{2})}$

$|a| = \frac{1}{2}$

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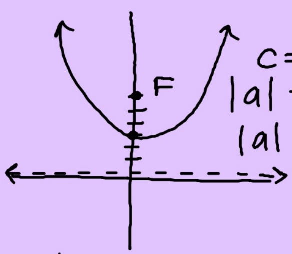
Write the equation of a parabola with F (0, 6) and directrix y = 0.



$c = 3$

$|a| = \frac{1}{4 \cdot 3}$

$|a| = \frac{1}{12}$

$y = \frac{1}{12}x^2 + 3$

A parabolic mirror has a focus that is located 4 in above the vertex of the mirror. Write an equation of the parabola that model the cross section of the mirror.

$c = 4$

$|a| = \frac{1}{4 \cdot 4}$

$|a| = \frac{1}{16}$

$y = \frac{1}{16}x^2$

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