10.4a Ellipses

Ellipse – is a set of points *P* in a plane such that the sum of the distances from P to two fixed points F_1 and F_2 is a given constant *k*.

Ellipse – Created by two foci (focus). The distances from the focus to the same point on the ellipse is constant.

$PF_1 + PF_2 = k$, where $k > F_1F_2$

Major Axis – Longer segment. Foci are on it.

Minor Axis – Smaller segment. Perpendicular to the major axis.

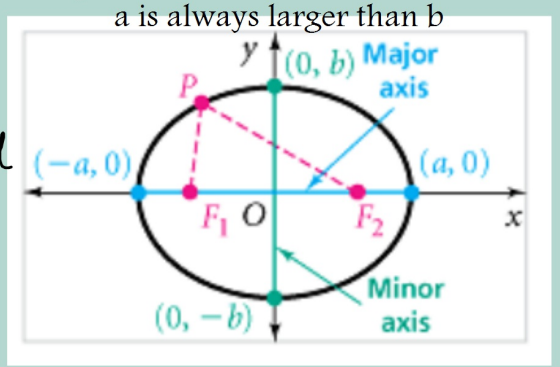
Vertices – Endpoints of the major axis

Co-vertices – Endpoints of the minor axis.

The planets follow elliptical, not circular, orbits around the sun

HORIZONTAL ELLIPSE: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2}$ (standard form).

a is always larger than b



Center: (h, k)

Vertices: (±a, 0)

Major Axis: horizontal

Co-Vertices: (0, ±b)

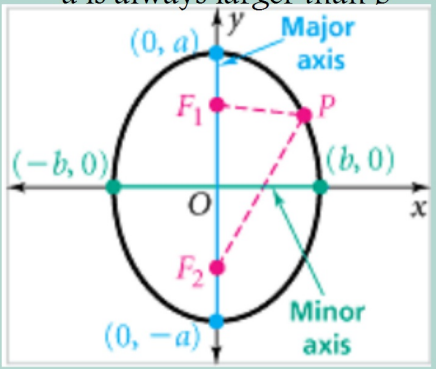
Minor axis: vertical

Foci: (±c, 0)

$c^2 = a^2 - b^2$

VERTICAL ELLIPSE: $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2}$ (standard form).

a is always larger than b



Center: (h, k)

Vertices: (0, ±a)

Major Axis: vertical

Co-Vertices: (±b, 0)

Minor axis: horizontal

Foci: (0, ±c)

Write the equation of the ellipse in standard form.

Vertices at (0, 5) and (0, −5), co-vertices at (2, 0) and (−2, 0) and the center at the origin.

vertical

$a = 5 \quad a^2 = 25$

$b = 2 \quad b^2 = 4$

center (0, 0)

$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$

Write the equation of the ellipse in standard form.

Vertex at $(-4, 0)$ and co-vertex at $(0, 3)$ and the center at the origin.

$a = 4 \quad a^2 = 16$

$b = 3 \quad b^2 = 9$

horizontal

center $(0,0)$

$\frac{x^2}{16} + \frac{y^2}{9} = 1$

Find an equation of an ellipse centered at the origin that is 12 units wide and 30 units high.

vertical

major axis $30 = 2a$

$15 = a \; ; \; a^2 = 225$

minor axis horizontal $12 = 2b$

$6 = b \; ; \; b^2 = 36$

$\frac{x^2}{36} + \frac{y^2}{225} = 1$

Find an equation of an ellipse centered at the origin that is 20 units wide and 10 units high.

$2a = 20$

$a = 10$

$a^2 = 100$

$2b = 10$

$b = 5$

$b^2 = 25$

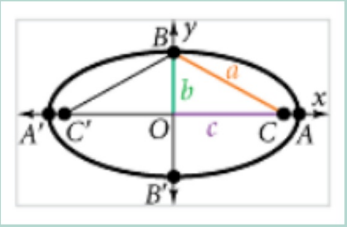
horizontal

$\frac{x^2}{100} + \frac{y^2}{25} = 1$

Foci of an ellipse are on the major axis

Foci are c units from the center

Use $c^2 = a^2 - b^2$ to find the foci.



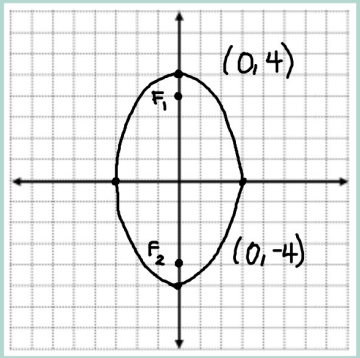
The sun is at a focus, not at the center of Earth's orbit around the sun.

Find the foci for the equation, then graph.

$\frac{x^2}{9} + \frac{y^2}{25} = 1$ $c^2 = a^2 - b^2$

$c^2 = 25 - 9$

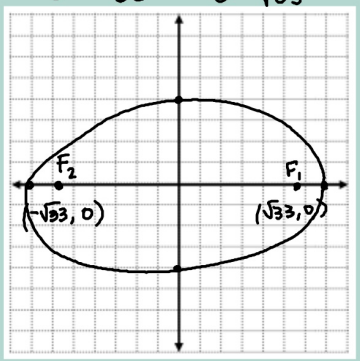
$c^2 = 16$ $c = \pm 4$



$\frac{16x^2}{784} + \frac{49y^2}{784} = \frac{784}{784}$ $\frac{x^2}{49} + \frac{y^2}{16} = 1$

$c^2 = 49 - 16$

$c^2 = 33$ $c = \sqrt{33}$



Write an equation of the ellipse with foci at $(0, \pm\sqrt{17})$ and co-vertices at $(\pm 8, 0)$.

vertical

$\frac{x^2}{64} + \frac{y^2}{81} = 1$

$c^2 = a^2 - b^2$

$17 = a^2 - 64$

$+64 \quad +64$

$81 = a^2$

Write an equation of the ellipse with foci at $(0, \pm 4)$ and co-vertices at $(\pm 2, 0)$

$b = 2$

$b^2 = 4$ $\frac{x^2}{4} + \frac{y^2}{20} = 1$

$c^2 = a^2 - b^2$

$4^2 = a^2 - 4$

$16 = a^2 - 4$

$20 = a^2$

homework:

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