10.5a Hyperbolas Centered at (0, 0)

HYPERBOLA – Created by two foci. Set of points where the distance between distances from some point P to the two foci is a constant k.

$|PF_1 - PF_2| = k$, where $k < F_1F_2$

TRANSVERSE AXIS (= 2a) – The segment that lies on the line containing the foci and has endpoints on the hyperbola.

VERTICES – The endpoints of the transverse axis and the start of the hyperbola.

ASYMPTOTES – A line that the graph approaches, but never crosses. Hyperbolas have two asymptotes that help us graph them.

Horizontal Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (standard form)

(a is always first)

Center: $(0, 0)$

Vertices: $(\pm a, 0)$

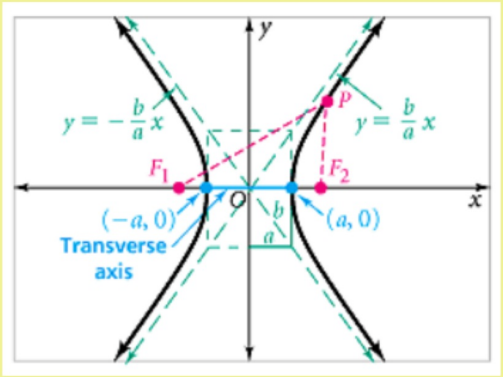
Asymptotes: $y = \pm \frac{b}{a}x$

x-intercepts: $(\pm a, 0)$

y-intercepts: none

Foci: $c^2 = a^2 + b^2$

$(\pm c, 0)$



Vertical Hyperbola: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ (standard form)

(a is always first)

Center: $(0, 0)$

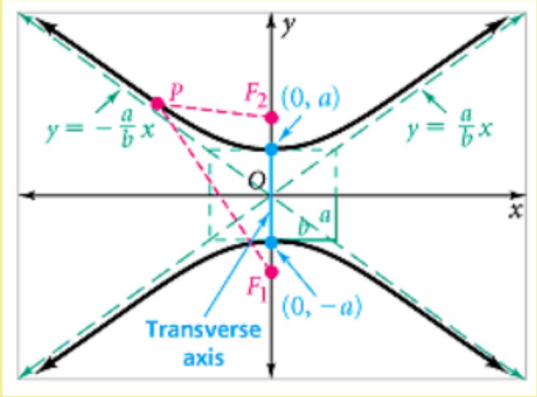
Vertices: $(0, \pm a)$

Asymptotes: $y = \pm \frac{a}{b}x$

x-intercepts: none

y-intercepts: $(0, \pm a)$

Foci: $(0, \pm c)$



To graph a hyperbola, use the standard form to find the values of *a* and *b*.

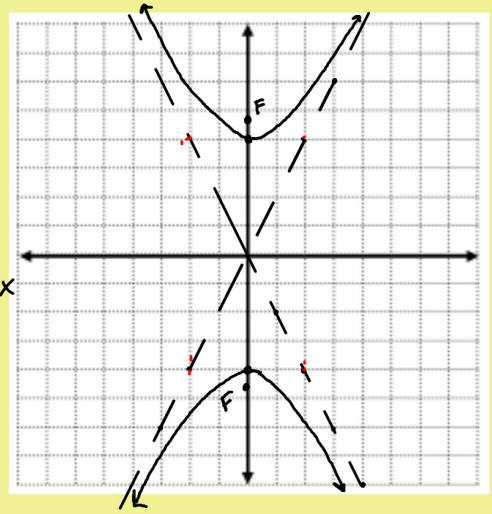
Option 1: Use *a* and *b* to find and graph the vertices and draw a central rectangle to guide the graph.

Draw the asymptotes through the central rectangle.

Option 2: Use the slopes of the asymptotes to draw the asymptotes through the center of the hyperbola.

Then draw the branches of the hyperbola through the vertices so they approach the asymptotes.

Graph $\frac{y^2}{16} - \frac{x^2}{4} = 1$



vertical

vertices $(0, \pm 4)$

center $(0, 0)$

asymptotes $y = \pm \frac{4}{2}x$

$y = \pm 2x$

$a = 4$

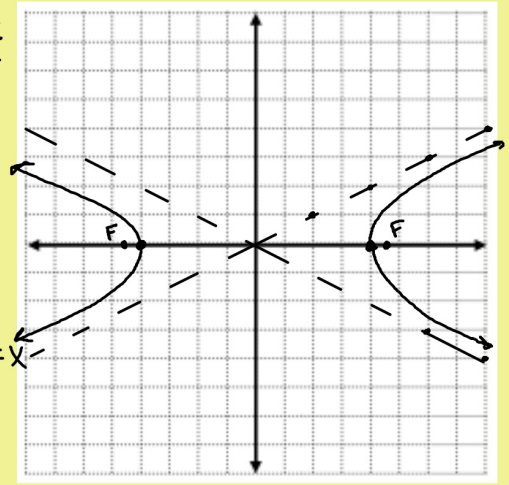
$b = 2$

foci $c^2 = 16 + 4$

$(0, \pm 2\sqrt{5})$ $c^2 = 20$

$c = \pm 2\sqrt{5}$

Graph $\frac{4x^2}{64} - \frac{16y^2}{64} = \frac{64}{64}$



$\frac{x^2}{16} - \frac{y^2}{4} = 1$

center $(0, 0)$

vertices $(\pm 4, 0)$

$a = 4$ $b = 2$

asymptotes $y = \pm \frac{2}{4}x$

$y = \pm \frac{1}{2}x$

foci $c^2 = 16 + 4$

$c^2 = 20$

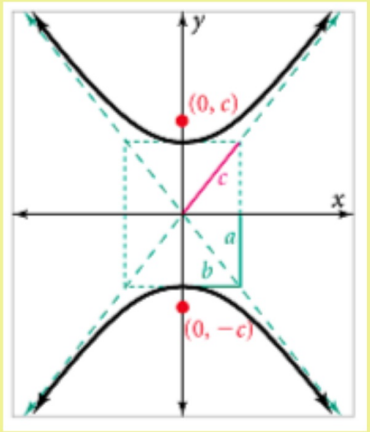
$c = \pm 2\sqrt{5}$ $(\pm 2\sqrt{5}, 0)$

Foci of a hyperbola are on the transverse axis.

Foci are c units from the center.

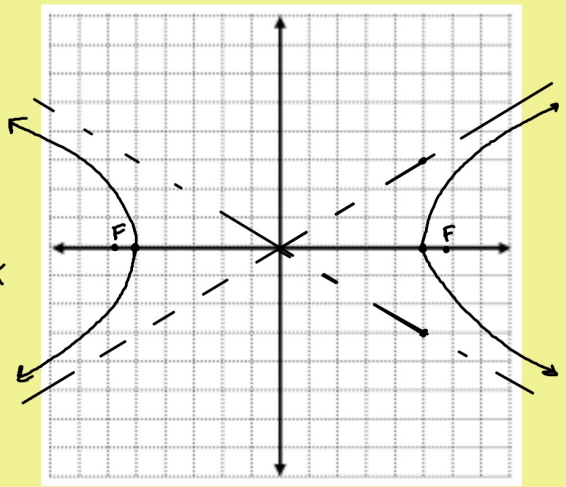
Use $c^2 = a^2 + b^2$

Distance between the foci is $2c$



Find the foci and graph.

$\frac{x^2}{25} - \frac{y^2}{9} = 1$



center $(0, 0)$

vertices $(\pm 5, 0)$

$a = 5$ $b = 3$

asymptotes $y = \pm \frac{3}{5}x$

foci $c^2 = a^2 + b^2$

$c^2 = 25 + 9$

$c^2 = 34$

$c = \pm\sqrt{34}$

$(\pm\sqrt{34}, 0)$

Page 9

Find the foci and graph.

$\frac{y^2}{4} - \frac{x^2}{9} = 1$

center $(0,0)$

vertices $(0, \pm 2)$

$a = 2 \quad b = 3$

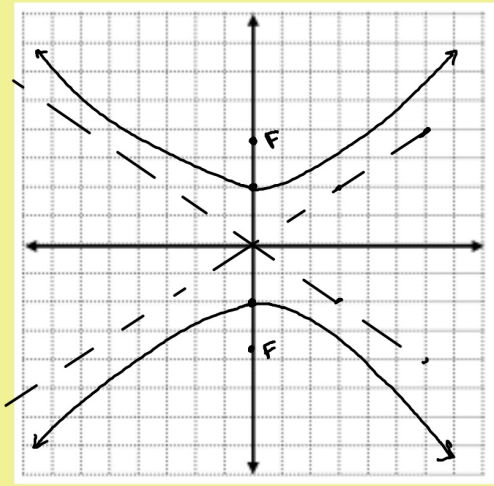
asymptotes $y = \pm \frac{2}{3}x$

foci $c^2 = a^2 + b^2$

$c^2 = 4 + 9$

$c = \sqrt{13}$

$(0, \pm\sqrt{13})$



Page 10

Write the equation of the hyperbola with the given values:

foci $(\pm 6, 0)$ vertices $(\pm 4, 0)$ $\quad a = 4 \quad a^2 = 16$

$$\frac{x^2}{16} - \frac{y^2}{20} = 1$$

$c^2 = a^2 + b^2$

$36 = 16 + b^2$

$20 = b^2$

Page 11

Write the equation of the hyperbola with the given values:

foci $(0, \pm 13)$ vertices $(0, \pm 5)$ $\quad a = 5 \quad a^2 = 25$

$c = 13 \quad c^2 = 169$

$$\frac{y^2}{25} - \frac{x^2}{144} = 1$$

$169 = 25 + b^2$

$144 = b^2$

Page 12

homework:

page 566 # 2-18 even, 19, 20, 23, 24, 42, 44