

### 11.3 Geometric Series

Geometric Sequence - The ratio between consecutive terms is constant.

Common Ratio - The ratio.

Unlike in an arithmetic sequence, the difference between consecutive terms varies.

Identify each sequence as arithmetic, geometric, or neither. Then give the common difference or common ratio.

1, -6, 36, -216, ...

geometric  $r = -6$

2, 4, 6, 8, ...

arithmetic  $d = 2$

6, 24, 96, 384, ...

geometric  $r = 4$

8, 20, 32, 44, ...

arithmetic  $d = 12$

### Geometric Sequence Formulas

In these formulas,  $a_n$  is the  $n$ th term,  $a_1$  is the first term,  $n$  is the number of the term, and  $r$  is the common ratio.

**Recursive Formula**

**Explicit Formula**

$$a_1 = \text{a given value}, a_n = a_{n-1} \cdot r$$

$$a_n = a_1 \cdot r^{n-1}$$

Write the recursive & explicit formulas, then find the 19th term.

11, 33, 99, 297, ...

E:

$$R: a_1 = 11 \\ a_n = a_{n-1} (3)$$

$$a_n = 11 \cdot 3^{n-1}$$

$$a_{19} = 11 \cdot 3^{19-1}$$

$$= 4,261,625,379$$

Write the recursive & explicit formulas, then find the 17th term.

3, 12, 48, 192, ...

$$R: a_1 = 3 \\ a_n = a_{n-1} (4)$$

$$E: a_n = 3(4)^{n-1}$$

$$a_{17} = 3 \cdot 4^{17-1}$$

$$= 3 \cdot 4^{16}$$

$$= 12,884,901,890$$

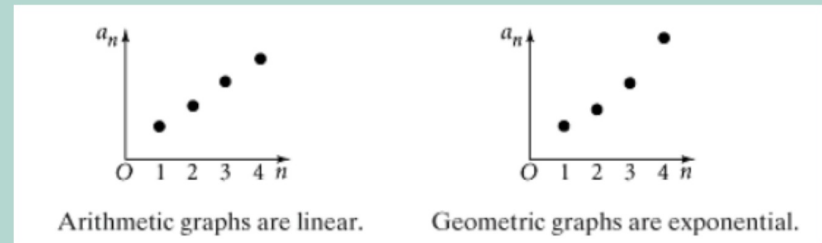
Suppose you want to enlarge a photo to 120% of its original size. A photo has a length of 10 cm. Find the length of the photo after 5 enlargements at 120%.

$$a_n = 10 \cdot 1.2^{n-1}$$

$$a_5 = 10 \cdot 1.2^{5-1}$$

$$a_5 = 20.736 \text{ cm}$$

The graphs of arithmetic and geometric sequences have different shapes.



Geometric Mean of any two positive numbers is the positive square root of the product of the two numbers.

$$\text{Geometric Mean} = \sqrt{\text{product of the two numbers}}$$

You can use the geometric mean to find a missing term of a geometric sequence.

Find the missing term. 20, 40, 80

$$\sqrt{20 \cdot 80} = \sqrt{1600} = 40$$

Find the missing term. 3, 7.5, 18.75

$$\sqrt{3 \cdot 18.75} = \sqrt{56.25} = 7.5$$

You can use the geometric mean to find a missing term of a geometric sequence.

Find the missing term. 28, 378, 5103

$$\sqrt{28 \cdot 5103} = \sqrt{142,884} = 378$$

A family purchased a home for \$150,000. Two years later the home was valued at \$188,160. If the value of the home is increasing geometrically, how much was the home worth after one year?

$$\sqrt{150,000 \cdot 188,160}$$

$$\sqrt{28,224,000,000}$$

$$\$168,000$$

Write a recursive and an explicit formula for the sequence.

5, 15, 45, 135, ...

$$R: \begin{array}{l} a_1 = 5 \\ a_n = a_{n-1}(3) \end{array} \quad E: a_n = 5(3)^{n-1}$$

Write a recursive and an explicit formula for the sequence.

125, -25, 5, -1, ...

$$R: \begin{array}{l} a_1 = 125 \\ a_n = a_{n-1}\left(-\frac{1}{5}\right) \end{array} \quad E: a_n = 125\left(-\frac{1}{5}\right)^{n-1}$$