11.4 Arithmetic Series

Series – The sum of all the numbers in a sequence.

Sequence: 1, 4, 9, 16, ...

Series: 1 + 4 + 9 + 16 + ...

Arithmetic Series – an arithmetic sequence added together.

Finite Sequences and Series – have an ending.

Infinite Sequence and Series – continue without end

Finite Sequence 6, 9, 12, 15, 18	Finite Series 6 + 9 + 12 + 15 + 18
Infinite Sequence 3, 7, 11, 15, ...	Infinite Series 3 + 7 + 11 + 15 + ...

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Write the sequence as a series, then evaluate.

5, 9, 13, 17, 21, 25, 29

5 + 9 + 13 + 17 + 21 + 25 + 29

= 119

100, 125, 150, 175, 200, 225

100 + 125 + 150 + 175 + 200 + 225

= 975

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Sum of a Finite Arithmetic Series

a_1 is the first term, a_n is the last term and n is the number of terms.

$$S_n = \frac{n}{2}(a_1 + a_n)$$

The sequence has 20 terms, find the sum of the series.

3, 10, 17, ..., 129, 136

$S_{20} = \frac{20}{2}(3 + 136)$

$= 10(139)$

$= 1390$

The sequence has 15 terms, find the sum of the series.

2, 4, 6, 8, ..., 26, 28, 30

$S_{15} = \frac{15}{2}(2 + 30)$

$= 7.5(32)$

$= 240$

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Evaluate the series to the 100th term

2 + 3 + 4 + 5 + ...

$a_{100} = 2 + (100 - 1)(1)$

$a_{100} = 2 + 100 - 1 = 1 + 100$

$= 1 + 100 = 101$

$S_{100} = \frac{100}{2}(2 + 101) = 50(103) = 5150$

Evaluate the series to the 1000th term.

1500 + 1499 + 1498 + 1497 + ...

$a_{1000} = 1500 + (1000 - 1)(-1)$

$= 1500 + (999)(-1)$

$= 1500 - 999$

$= 501$

$S_{1000} = \frac{1000}{2}(1500 + 501) = 500(2001)$

$= 1{,}000{,}500$

Evaluate the series to the 50th term.

5 + 8 + 11 + 14 + ...

$a_{50} = 5 + (50 - 1)3$

$= 5 + 49(3)$

$= 152$

$S_{50} = \frac{50}{2}(5 + 152)$

$= 3925$

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You can use the summation symbol Σ to write a series. Then you can use limits to indicate how many terms you are adding.

Limits – are the least and greatest integral values of n.

Upper Limit, greatest value of n

Explicit Formula for the sequence

$$\sum_{n=1}^{3}(5n+1)$$

Lower Limit, least value of n

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Use the summation notation to write the series for 27 terms.

$3+6+9+\ldots$

$a_n = 3+(n-1)3$
$a_n = 3+3n-3$
$a_n = 3n$

$$\sum_{n=1}^{27}(3n)$$

Use the summation notation to write the series for 9 terms

$3+8+13+18+\ldots$

$a_n = 3+(n-1)5$
$a_n = 3+5n-5$
$a_n = 5n-2$

$$\sum_{n=1}^{9}(5n-2)$$

Use summation notation to write the series $8+16+24+\ldots$ for 50 terms

$a_n = 8+(n-1)8$
$a_n = 8+8n-8$
$a_n = 8n$

$$\sum_{n=1}^{50}(8n)$$

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Use the series $\sum_{n=1}^{4}(-2n+3)$

A) Find the number of terms in the series 4

B) Find the 1st and last terms

$a_1 = -2(1)+3$
$= -2+3$
$= 1$

$a_4 = -2(4)+3$
$= -8+3$
$= -5$

C) Evaluate the series.

$S_4 = \frac{4}{2}(1+(-5))$
$S_4 = 2(-4)$
$S_4 = -8$

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Use the series $\sum_{n=2}^{5} n^2$

A) Find the number of terms in the series 4

B) Find the 1st and last terms

$a_2 = 2^2 = 4$
$a_5 = 5^2 = 25$

C) Evaluate the series.

$S_4 = \frac{4}{2}(4+25)$
$S_4 = 2(29)$
$S_4 = 58$

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Use the series $\sum_{n=1}^{4}\left(\frac{1}{2}n+1\right)$

A) Find the number of terms in the series 4

B) Find the 1st and last terms $a_1 = \frac{1}{2}(1)+1 = \frac{3}{2}$

$a_4 = \frac{1}{2}(4)+1 = 3$

C) Evaluate the series.

$S_4 = \frac{4}{2}\left(\frac{3}{2}+3\right)$

$= 2\left(\frac{9}{2}\right)$

$= 9$

Use the series $\sum_{n=1}^{10}(n-3)$

A) Find the number of terms in the series 10

B) Find the 1st and last terms $a_1 = 1-3 = -2$

$a_{10} = 10-3 = 7$

C) Evaluate the series.

$S_{10} = \frac{10}{2}(-2+7)$

$= 5(5)$

$= 25$