11-5 Geometric Series

Geometric Series - The sum of the terms of a geometric sequence.

Sum of a Finite Geometric Series: $S_n = \frac{a_1(1-r^n)}{1-r}$

a_1 is the first term, r is the common ratio and n is the number of terms.

Identify a_1, r, and n for each series. Then evaluate each series.

$5 + 15 + 45 + 135 + 405 + 1215$

$a_1 = 5$ $\quad S_6 = \frac{5(1-3^6)}{1-3}$

$r = 3$

$n = 6$ $\quad S_6 = \frac{5(-728)}{-2} = 1820$

$-45 + 135 - 405 + 1215 - 3645$

$a_1 = -45$ $\quad S_5 = \frac{-45(1-(-3)^5)}{1-(-3)}$

$r = -3$

$n = 5$

$= \frac{-45(1+243)}{4} = -2745$

In some cases you can evaluate an infinite geometric series.

Converges - When a ratio makes the numbers get smaller and smaller. Gets closer and closer to the sum.

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

· $|r| < 1$

· Has a sum

Diverges - when a ratio makes the numbers get larger. Approaches no limit.

$\sum_{n=1}^{\infty} 5(2)^{n-1}$

· $|r| \geq 1$

· Does not have a sum

Decide whether each infinite geometric series diverges or converges. State whether the series has a sum.

$2 + 6 + 18 + \ldots$

$r = 3$ diverges no sum

$\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$ Converges has a sum

$r = \frac{2}{3}$

$1 + \frac{1}{5} + \frac{1}{25} + \ldots$ converges has a sum

$r = \frac{1}{5}$

$4 + 8 + 16 + \ldots$ diverges no sum

$r = 2$

Sum of an Infinite Geometric Series: An infinite geometric series with $|r| < 1$ converges to the sum:

$$S = \frac{a_1}{1-r}$$

Find the sum of the converging series.

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

$a_1 = 1$ $\quad S = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$

$r = \frac{1}{2}$

$3 - \frac{3}{2} + \frac{3}{4} - \frac{3}{8} + \ldots$

$a_1 = 3$ $\quad S = \frac{3}{1-(-\frac{1}{2})} = \frac{3}{\frac{3}{2}} = 2.$

$r = -\frac{1}{2}$