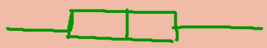
12-4 Standard Deviation

Measures of Variation – Describes how the data in a data set is spread out.

Range – Difference between the greatest and the least values.

Interquartile Range (IQR) – Difference between the third and first quartiles.



Outlier: To find an outlier take 1.5 times the interquartile range (IQR) and add it to the third quartile and subtract it from the first quartile. Anything not in between these is an outlier.

$Q_1 - 1.5\text{ IQR} \quad Q_3 + 1.5\text{ IQR}$

Example 1: Seventeen women qualified for the 2002 U.S. Women's Alpine Ski Team. Find the range, interquartile range, and any outliers of their ages at the time of qualification:

24, 30, 29, 21, 22, 22, 28, 21, 16, 17, 25, 22, 21, 18, 19, 18, 19.

16 17 18 18 | 19 19 21 21 $\boxed{21}$ 22 22 22 24 | 25 28 29 30

18.5 Q_1 median 24.5 Q_3

range: 30 − 16 = 14

IQR: 24.5 − 18.5 = 6

1.5(IQR): 1.5(6) = 9

no one is an outlier

$$\begin{array}{r} 18.5 \\ -\ 9 \\ \hline <9.5 \end{array} \qquad \begin{array}{r} 24.5 \\ +\ 9 \\ \hline >33.5 \end{array}$$

outliers

Standard Deviation – A measure of how each value in a data set varies, or deviates, from the mean. It is represented by the Greek letter σ (sigma).

Finding Standard Deviation

- Find the mean of the data set: $\bar{x}$
- Find the difference between each value and the mean: $x - \bar{x}$
- Square each difference: $(x - \bar{x})^2$
- Find the average (mean) of these squares: $\frac{\Sigma(x - \bar{x})^2}{n}$
- Take the square root to find the standard deviation: $\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}$

Example 2: Find the mean and the standard deviation for these values:

50, 60, 70, 80, 80, 90, 100, 110.

$$\frac{\Sigma}{n} = \frac{50+60+70+80+80+90+100+110}{8}$$

$\bar{x} = 80$

x	$\bar{x}$	$x-\bar{x}$	$(x-\bar{x})^2$
50	80	−30	900
60	80	−20	400
70	80	−10	100
80	80	0	0
80	80	0	0
90	80	10	100
100	80	20	400
110	80	30	900

$\sigma = \sqrt{\frac{2800}{8}}$

$\sigma = 18.708$

Standard deviation is like a custom-made measuring stick for the variation in a set of data. A small standard deviation (compared to actual data values) indicates that the data are clustered tightly around the mean. As the data become more spread out, the standard deviation increases.

Every value falls within some number of standard deviations of the mean.

When a value falls within one standard deviation of the mean, it is in the range of values from one standard deviation below the mean to one standard deviation above. For example, if the mean is 50 and the standard deviation is 10, then a value x within one standard deviation of the mean must be in the range

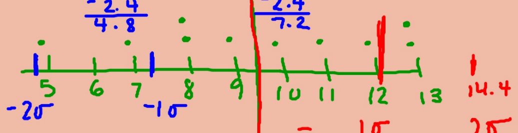
$$40 \le x \le 60.$$

Example 3: The number of points that Darden scored in each of 11 basketball games is listed below. Within how many standard deviations of the mean do all of the values fall? What can Darden's coach do with this information?

$\bar{x} = 9.6$
$\sigma = 2.4$

8, 12, 13, 10, 7, 5, 10, 9, 13, 11, 8

$7.2 - 2.4 = 4.8$ $\quad$ $9.6 - 2.4 = 7.2$



$9.6 = \bar{x}$, $+2.4$, 12 $\quad$ $12 + 2.4 = 14.4$

all values are within 2 SD.

95% of the time, he score 5-13 points

z-score – The number of standard deviations that a value is from the mean.

$$\text{z-score} = \frac{\text{value} - \text{mean}}{\text{standard deviation}} = \frac{x - \bar{x}}{\sigma}$$

Example 4:

A. A set of values has a mean of 22 ($\bar{x}$) and a standard deviation of 5 (σ). Find the z-score for a value of 30 (x).

$$\frac{30-22}{5} = 1.6$$

B. A set of values has a mean of 85 ($\bar{x}$) and a standard deviation of 6 (σ). Find the value (x) that has a z-score of 2.5 (z).

$$2.5 = \frac{x-85}{6}$$

$$15 = x - 85 \qquad x = 100$$

Variance – Standard Deviation squared. σ^2

Example 6: Find the variance for Examples 2 (σ = 18.708) and 4 (σ = 1.6).

349.99

2.56

H.W. p. 660, 1-25 all

1 and 5 find standard deviation with formula not calculator!!!