13.3 Radian Measure

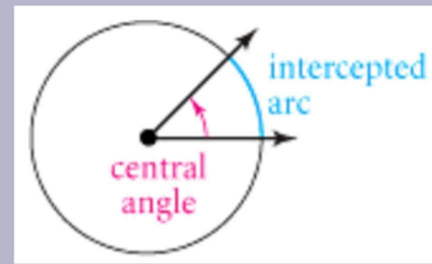
A central angle of a circle is an angle with a vertex at the center of the circle.

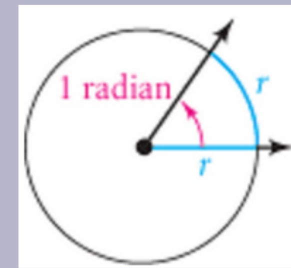
An intercepted arc is the portion of the circle with endpoints on the sides of the central angle and remaining points within the interior of the angle.

Angle-how far you turn or rotate.

Circumference- how far to "walk" along the edge of a circle. When the central angle intercepts an arc that has the same length as a radius of the circle, the measure of the angle is defined to be one radius.

Page 1





Because the circumference of a circle is 2πr, there are 2π radians in any circle. 360° = 2π Radians. 180° = π Radians.

$$\frac{d^o}{180} = \frac{r\,\text{radians}}{\pi\,\text{radians}}$$

Page 2

Use a proportion for each conversion. ~~NO DECIMALS unless the book gives you one.~~

85° to radians

$$\frac{85}{180} = \frac{r}{\pi}$$

$$\frac{85\pi}{180} = \frac{180r}{180}$$

1.48 radians

2.5 radians to degrees

$$\frac{d}{180} = \frac{2.5}{\pi}$$

$$\frac{\pi d}{\pi} = \frac{450}{\pi}$$

$d = 143.2°$

$\frac{13\pi}{6}$ to degrees

$$\frac{d}{180} = \frac{13\cancel{\pi}/6}{\cancel{\pi}}$$

$$\frac{d}{180} = \frac{13}{6}$$

390°

Page 3

To convert degrees to radians, multiply by $\frac{\pi}{180°}$

To convert radians to degrees, multiply by $\frac{180°}{\pi}$

Use dimensional analysis to convert each degree measure from degrees to radians or from radians to degrees. (Express radian measures in terms of π.

$\frac{\pi}{2}$ radians

$$\frac{\cancel{\pi}}{\cancel{2}_1} \cdot \frac{\cancel{180}^{90}}{\cancel{\pi}}$$

90°

225°

$$\cancel{225}^5 \cdot \frac{\pi}{\cancel{180°}_4}$$

$$\frac{5\pi}{4}$$

2 radians

$$2 \cdot \frac{180}{\pi}$$

$$\frac{360}{\pi} = 114.6$$

150°

$$\cancel{150}^5 \cdot \frac{\pi}{\cancel{180}_6}$$

$$\frac{5\pi}{6}$$

Page 4

You can find the sine and cosine of angles in radian measure by first converting the radian measure to degrees and then using the unit circle. Find the exact values of the following:

$\cos(\frac{\pi}{4})$ and $\sin(\frac{\pi}{4})$ $\quad \cos(45)\ \frac{\sqrt{2}}{2} \quad \sin(45)\ \frac{\sqrt{2}}{2}$

$\cos(\frac{\pi}{3})$ and $\sin(\frac{\pi}{3})$ $\quad \cos(60)\ \frac{1}{2} \quad \sin(60)\ \frac{\sqrt{3}}{2}$

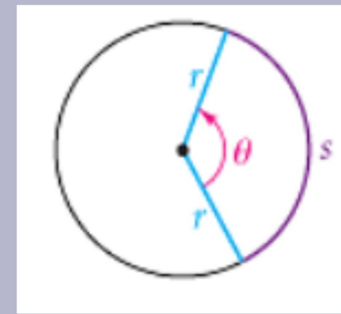
$\cos(\frac{7\pi}{6})$ and $\sin(\frac{7\pi}{6})$ $\quad \cos(210)\ -\frac{\sqrt{3}}{2} \quad \sin(210)\ -\frac{1}{2}$

Page 5

Length of an Intercepted Arc

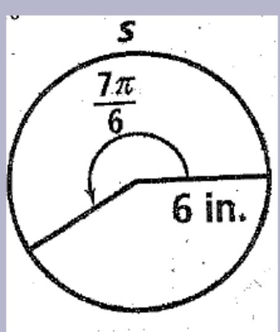
For a circle of radius r and a central angle of measure θ (in radians), MODE

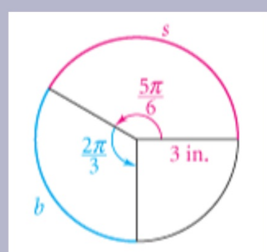
the length s of the intercepted arc is $s = r\theta$



Page 6

Use the circle to find the length of s and b to the nearest tenth.





$s = \cancel{3}\left(\frac{5\pi}{2\cancel{6}}\right)$

$s = \frac{5\pi}{2}$

$s = 7.9$ in

$s = r\theta$

$s = \cancel{6} \cdot \frac{7\pi}{\cancel{6}}$

$s = 7\pi$

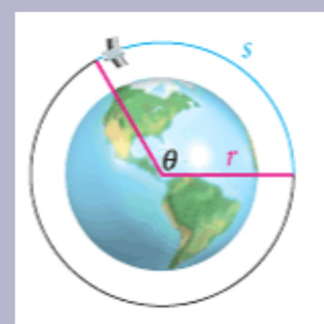
$s = 22$ in.

$b = \cancel{3}\left(\frac{2\pi}{\cancel{3}}\right)$

$b = 6.28$ in

Page 7

A weather satellite in a circular orbit around Earth completes one orbit every 3 hours. The radius of Earth is about 6400 km, and the satellite orbits 2600 km above Earth's surface. How far does the satellite travel in one hour?



$6400 + 2600 = 9000$ km

$2\pi = 3$ hrs.

$\frac{2\pi}{3} = 1$ hr.

$S = 9000\left(\frac{2\pi}{3}\right)$

$S = 3000(2\pi)$

$S = 6000\pi = 18849.6$ km

Page 8

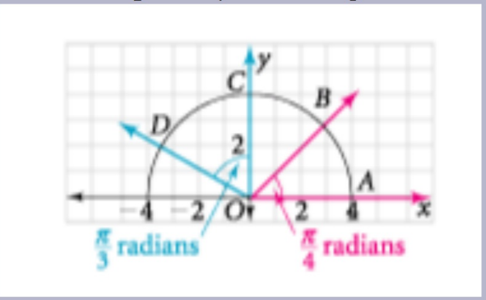
Find the length of the arc intercepted by each angle.

A. $\angle AOB$

B. $\angle COD$

C. $\angle AOC$

D. $\angle AOD$



A. $s = 4\left(\frac{\pi}{4}\right) = \pi = 3.14$

B. $s = 4\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} = 4.19$

C. $s = 4\left(\frac{\pi}{2}\right) = 2\pi = 6.28$

D. $s = 4\left(\frac{5\pi}{6}\right) = \frac{10\pi}{3}$

$= 10.47$

homework:

page 715 # 1-27, 1-6: radian not degree