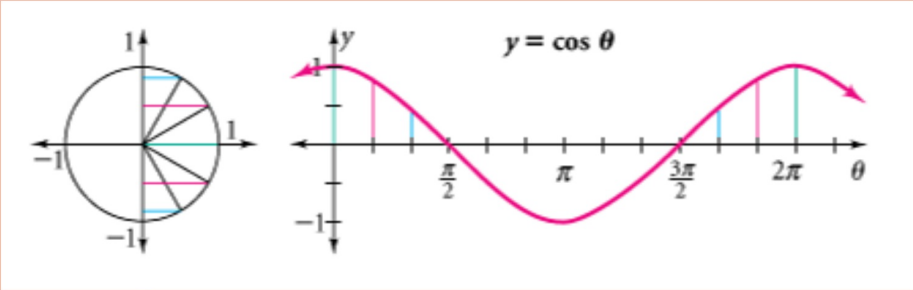
13.5 The Cosine Function

Cosine Function – $y = \cos\theta$, matches the measure of θ of an angle in standard position with the x-coordinate of a point on the unit circle.

Use the graph below. Find the domain, period, range, and amplitude.



d: $\mathbb{R}$ range $-1 \le y \le 1$ period: 2π amp: 1

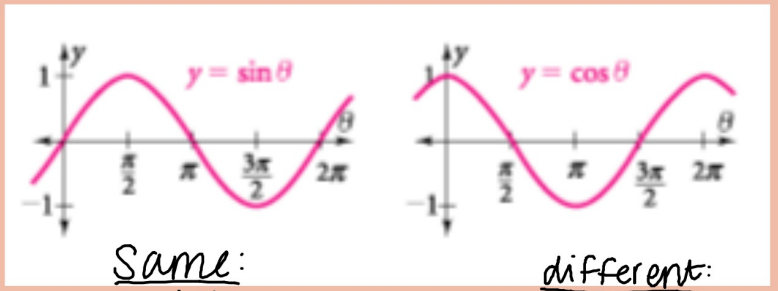
Examine the cycle of the cosine function in the interval from 0 to 2π. Where in the cycle does the maximum value occur? Where does the minimum value occur? Where do the zeros occur?

max 0, 2π

min π

zeros $\frac{\pi}{2}$, $\frac{3\pi}{2}$

Use the graphs below. How are the graphs of the sine and cosine functions alike? How are they different?



Same:
periodic
same domain & range
period 2π
amp = 1

different:
sin's max/min are cos's zeros. cos's max/min are sin's zeros.
sin shifts left $\pi/2$ = cos

Properties of Cosine Function

Suppose $y = a\cos b\theta$, with $a \neq 0$, $b > 0$, and θ in radians.

- $|a|$ is the amplitude of the funciton
- b is the number of cycles in the interval from 0 to 2π
- $\frac{2\pi}{b}$ is the period of the function.

To graph a cosine function, locate five points equally spaced through one cycle to sketch a cosine curve.

For $a > 0$, this five-point pattern is ***max-zero-min-zero-max***

For $a < 0$, this five-point pattern is *min-zero-max-zero-min*

Graph one cycle $y = 2\cos\pi\theta$

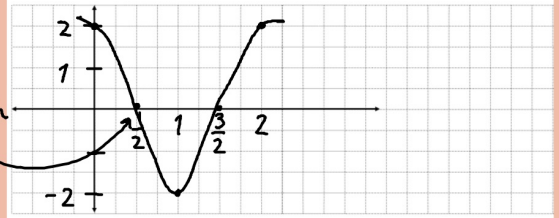
Amplitude = 2

Period = $\frac{2\pi}{b} = \frac{2\pi}{\pi} = 2$

Each = 2 divided into quarters = $\frac{1}{2}$ each

Flip = no (2 is positive)

Range = $-2 \le y \le 2$



Graph one cycle $y = -\frac{1}{2}\cos 2\theta$

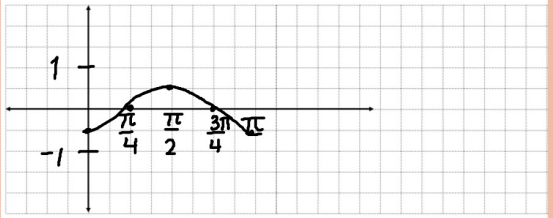
Amplitude = $-\frac{1}{2}$

Period = $\frac{2\pi}{2} = \pi$

Each = $\frac{\pi}{4}$

Flip = yes ($-\frac{1}{2}$ is neg.)

Range = $-\frac{1}{2} \le y \le \frac{1}{2}$



Graph one cycle $y = 3\cos\frac{\pi}{2}\theta$

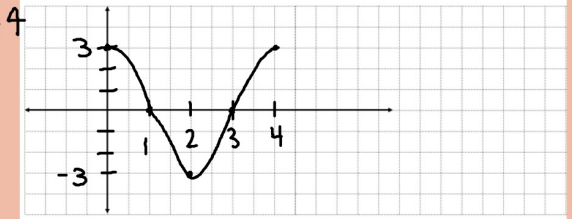
Amplitude = 3

Period = $\frac{2\pi}{\pi/2} = 2\pi \cdot \frac{2}{\pi} = 4$

Each = 1

Flip = no

Range = $-3 \le y \le 3$



Write a cosine function for each description. Choose a > 0.

Amplitude 4, period 6π

$\frac{2\pi}{b}$ = period

$\frac{2\pi}{b} = 6\pi$

$2\pi = 6\pi b$

$\frac{2\pi}{6} = \frac{6\pi b}{6}$

$\frac{1}{3} = b$

$y = 4\cos\frac{1}{3}\theta$

Amplitude 2.5, period 8

$\frac{2\pi}{b}$ = period

$\frac{2\pi}{b} = 8$

$2\pi = 8b$

$\frac{2\pi}{8} = \frac{8b}{8}$

$\frac{\pi}{4} = b$

$y = 2.5\cos\frac{\pi}{4}\theta$

You can solve an equation by graphing to find an exact location along a sine or cosine curve. Use the graphing calculator, in RADIANS, and find the intersections. Solve each equation in the interval from 0 to 2π. Round to the nearest hundredths.

$3\cos 2\theta = -2$ {1.2, 2, 4.3, 5.1}

$-2\cos\theta = 1.2$ {2.2, 4.1}

$-3\sin 2\theta = 1.5$ {1.8, 2.9, 5.0, 6}

MODE → RADIANS
$Y_1 = 3\cos 2X$
$Y_2 = -2$
GRAPH
2nd CALC → intersection
ENTER, ENTER, move cursor by intersection, ENTER.
Repeat

homework
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