

13.8 Reciprocal Trigonometric Functions

Cosecant, Secant and Cotangent Functions

The cosecant (csc), secant (sec), and cotangent (cot) functions are defined as reciprocals. Their domains include all real numbers except those that make the denominator zero.

$$\csc\theta = \frac{1}{\sin\theta} \quad \sec\theta = \frac{1}{\cos\theta} \quad \cot\theta = \frac{1}{\tan\theta}$$

Evaluate each expression

Suppose $\sin\theta = \frac{15}{8}$ find $\csc\theta = \frac{8}{15}$

Suppose $\cos\theta = \frac{4}{5}$ find $\sec\theta = \frac{5}{4}$

Suppose $\tan\theta = \frac{5}{12}$ find $\cot\theta = \frac{12}{5}$

Suppose $\cos\theta = \frac{5}{13}$ find $\sec\theta = \frac{13}{5}$

Evaluate each expression

Find $\cot 55^\circ$ to the nearest hundredth $\tan 55^\circ = 1.43 \rightarrow \frac{1}{1.43} = .70$

cannot use \tan^{-1} or $\frac{1}{\tan 55^\circ} = .70$

Find $\csc 60^\circ$ to the nearest hundredth
(use $\sin 60^\circ$) 1.15

Find $\sec 35^\circ$ to the nearest hundredth
(use $\cos 35^\circ$) 1.22

Find $\csc 45^\circ$ to the nearest hundredth
(use $\sin 45^\circ$) 1.41

You can use what you know about the unit circle to find exact values for reciprocal trigonometric functions.

Find the exact value.

$$\begin{array}{llll} \sec 60^\circ & \cot 45^\circ & \csc 120^\circ & \csc 30^\circ \\ \cos 60^\circ = \frac{1}{2} & \tan 45^\circ = 1 & \sin 120^\circ = \frac{\sqrt{3}}{2} & \sin 30^\circ = \frac{1}{2} \\ \sec 60^\circ = 2 & \cot 45^\circ = 1 & \csc 120^\circ = \frac{2}{\sqrt{3}\sqrt{3}} = \frac{2\sqrt{3}}{3} & \csc 30^\circ = 2 \end{array}$$

Each angle is now given in radians. Round to the nearest thousandths.

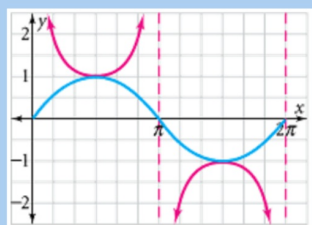
$$\begin{array}{llll} \csc(-1.5) & \sec 2 & \cot \pi & \cot \frac{\pi}{2} \\ \sin(-1.5) = -.997 & \frac{1}{\cos 2} = -2.403 & \tan \pi = -1 & \tan \frac{\pi}{2} = \frac{1}{0} \\ \frac{1}{\sin(-1.5)} = -1.003 & & \cot \pi = \frac{-1}{0} & \cot \frac{\pi}{2} = \frac{0}{1} = 0 \\ & & \text{undefined} & \end{array}$$

The graphs of trigonometric functions have asymptotes where their denominators are zero.

Sketch the graph of $y = \sin x$ and $y = \csc x$ in the interval from 0 to 2π .

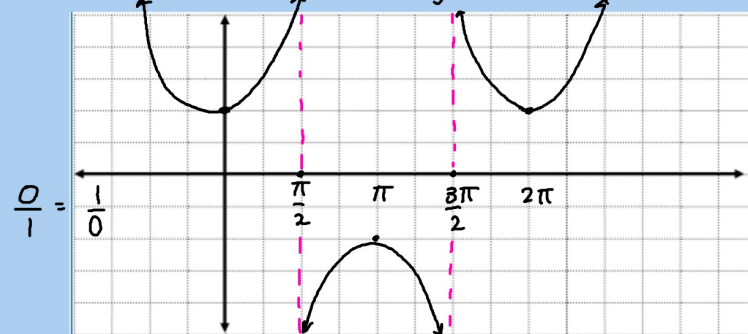
First, think about the table of values. Then plot the points and sketch the graph.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	0.5	0.9	1	0.9	0.5	0	-0.5	-0.9	-1	-0.9	-0.5	0
$\csc x$	■	2	1.2	1	1.2	2	■	-2	-1.2	-1	-1.2	-2	■



Graph $y = \sec x$ from 0 to 2π

$$y = \frac{1}{\cos x}$$



Graph $y = -\sec 2(\theta + \pi)$

$$\left(\frac{1}{\cos}\right)$$

$$a = 1$$

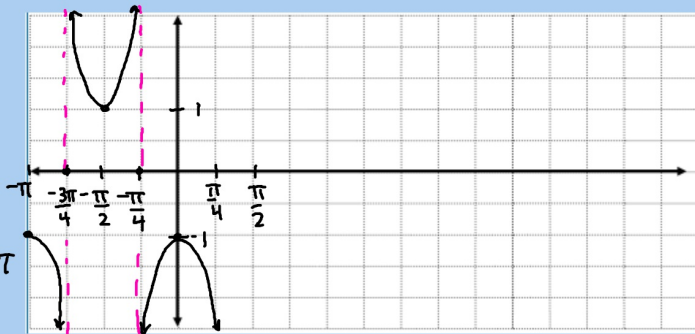
$$\text{period} = \pi$$

$$\text{each} = \frac{\pi}{4}$$

$$\text{flip} = \text{yes}$$

$$\text{phase} = \text{Left } \pi$$

$$\text{vertical} = \text{no shift}$$



Graph $y = \csc x$. Find $\csc 45^\circ$

Use your graphing calculator. Graph $y = \frac{1}{\sin x}$. Use the table.

$$1.41$$

homework:

page 752 # 3-39 x 3, 42-48 even