14.1 Trigonometric Identities

A Trigonometric Identity is a trigonometric equation that is true for all values except those for which the expressions on either side of the equal sign are undefined.

To verify an identity, you should transform one side of the equation until it is the same as the other side. This eliminates the possibility of introducing errors that can be caused by squaring both sides of an equation or multiplying both sides of an equation by an expression that equals 0. These are the errors that can introduce extraneous roots when solving equations.

It is sometimes helpful to write all the functions in terms of sine and cosine.

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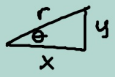

TRIGONOMETRIC IDENTITIES

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Tangent and Cotangent Identities $\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities $x^2 + y^2 = r^2$ ÷cos²θ ÷sin²θ

$\cos^2\theta + \sin^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

*There are additional identities

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Verify both tangent and cotangent identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\frac{\sin\theta}{\cos\theta} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{r} \cdot \frac{r}{x} = \frac{y}{x} = \tan\theta$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\frac{\cos\theta}{\sin\theta} = \frac{\frac{x}{r}}{\frac{y}{r}} = \frac{x}{r} \cdot \frac{r}{y} = \frac{x}{y} = \cot\theta$

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Verify $\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta + \sin^2\theta$

$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = \frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{x^2+y^2}{r^2} = \frac{r^2}{r^2} = 1$

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Verify $1+\tan^2\theta = \sec^2\theta$

$1+\tan^2\theta$

$1+\left(\frac{\sin\theta}{\cos\theta}\right)^2 = 1+\frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{\cos^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta$

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Verify $1+\cot^2\theta = \csc^2\theta$

$1+\cot^2\theta = 1+\frac{\cos^2\theta}{\sin^2\theta} = \frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}$

$= \frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} = \csc^2\theta$

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Verify $(\sin x+\cos x)^2 = 1+2\sin x\cos x$

$(\sin x+\cos x)^2 = \sin^2 x+2\sin x\cos x+\cos^2 x$

$a^2+2ab+b^2$

$= \sin^2 x+\cos^2 x+2\sin x\cos x$

$1+2\sin x\cos x$

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Verify each identity

$\sec^2\theta-\sec^2\theta\cos^2\theta = \tan^2\theta$

$\frac{1}{\cos^2\theta}-\frac{1}{\cos^2\theta}\cdot\cos^2\theta$

$\frac{1}{\cos^2\theta}-1$

$\sec^2\theta-1=\tan^2\theta$

$\frac{\sin\theta}{\tan\theta}+\frac{\cos\theta}{\cot\theta} = \sin\theta+\cos\theta$

$\frac{\sin\theta}{\frac{\sin\theta}{\cos\theta}}+\frac{\cos\theta}{\frac{\cos\theta}{\sin\theta}}$

$\sin\theta\cdot\frac{\cos\theta}{\sin\theta} \quad \cos\theta\cdot\frac{\sin\theta}{\cos\theta}$

$\cos\theta+\sin\theta$

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You can use the trigonometric identities to simplify trigonometric expressions.

Simplify each trigonometric expression.

$\sec\theta\cot\theta$

$= \frac{1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta}$

$= \frac{1}{\sin\theta}$

$= \csc\theta$

Pythagorean Id. subst.

$(1+\cot^2\theta)(\sec^2\theta - 1)$

$\csc^2\theta \cdot \tan^2\theta$

$\frac{1}{\sin^2\theta} \cdot \frac{\sin^2\theta}{\cos^2\theta}$

$\frac{1}{\cos^2\theta}$

$\sec^2\theta$

Simplify each trigonometric expression.

$\csc\theta\tan\theta$

$\frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta}$

$\frac{1}{\cos\theta}$

$\sec\theta$

homework:

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