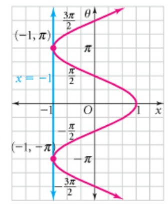
1 EXAMPLE Using a Graph to Find Angles With a Given Cosine

Inverse of $y = \cos\theta$



Use the graph of the inverse of $y = \cos\theta$ at the right.

a. Find the radian measures of the angles whose cosine is -1.

The line $x = -1$ intersects the graph at $(-1, \pi)$ and $(-1, -\pi)$. So the measures of two angles whose cosine is -1 are π and $-\pi$.

Other points of intersection are $(-1, \pm 3\pi)$, $(-1, \pm 5\pi)$, and so on. The measures of all the angles whose cosine is -1 can be written as $\pi + 2\pi n$, where n is any integer.

b. Find the radian measures of the angles θ whose cosine is 2.

The line $x = 2$ does not intersect the graph. 2 is not in the domain of the inverse of $y = \cos\theta$. There is no angle whose cosine is 2.

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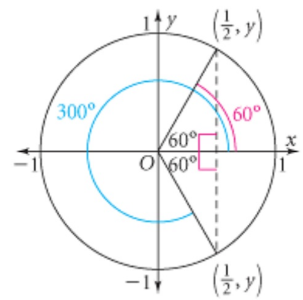
2 EXAMPLE Using a Unit Circle

Use a unit circle to find the degree measures of the angles whose cosine is $\frac{1}{2}$.

Draw a unit circle and mark the points on the circle that have x-coordinates of $\frac{1}{2}$. These points and the origin form 30°-60°-90° triangles. 60° and 300° are the measures of two angles whose cosine is $\frac{1}{2}$. All their coterminal angles also have a cosine of $\frac{1}{2}$. The measures of all the angles whose cosine is $\frac{1}{2}$ can be written as $60° + n \cdot 360°$ and $300° + n \cdot 360°$.



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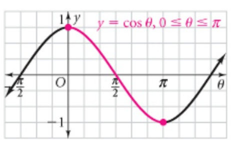
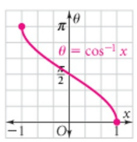
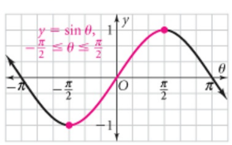
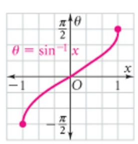
The domain of the cosine function can be restricted to $0 \le \theta \le \pi$ so that its inverse is a function. The inverse function is written $\theta = \cos^{-1} x$ and is read as "θ is the angle whose cosine is x."

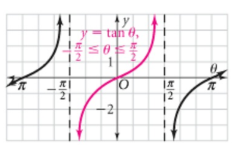


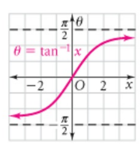


Similarly, the ranges of $y = \sin\theta$ and $y = \tan\theta$ are restricted to $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ to obtain the inverse functions $\theta = \sin^{-1} x$ and $\theta = \tan^{-1} x$.









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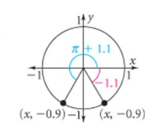
3 EXAMPLE Using a Calculator to Find the Inverse of Sine

Use a calculator and an inverse function to find the radian measures of all the angles whose sine is -0.9.

$\sin^{-1}(-0.9) \approx -1.12$ **Use a calculator.**

This angle is in Quadrant IV. The sine function is also negative in Quadrant III, as shown in the figure at the right. So $\pi + 1.12 \approx 4.26$ is another solution.



The radian measures of all the angles whose sine is -0.9 can be written as

$-1.12 + 2\pi n$ and $4.26 + 2\pi n$.

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4 EXAMPLE **Using a Calculator to Find the Inverse of Tangent**

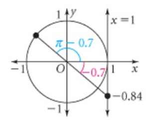
Use a calculator and an inverse function to find the measure in radians of all the angles whose tangent is -0.84.

$\tan^{-1}(-0.84) \approx -0.70$ **Use a calculator.**

The tangent function is also negative in Quadrant II, as shown in the figure at the right. So $\pi - 0.70 \approx 2.44$ is another solution.

The radian measures of all the angles whose tangent is -0.84 can be written as

$-0.70 + 2\pi n$ and $2.44 + 2\pi n$.



In contrast to trigonometric identities, most trigonometric equations are true for only certain values of the variable. Sometimes you have to solve trigonometric equations by factoring. Solve each equation for $0 \le \theta \le 2\pi$.

$3\sin\theta + 1 = \sin\theta$

$-\sin\theta \quad -\sin\theta$

$2\sin\theta + 1 = 0$

$-1 \quad -1$

$\frac{2\sin\theta}{2} = \frac{-1}{2}$

$\sin\theta = -\frac{1}{2}$

QIII & QIV

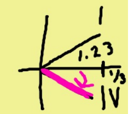
$\frac{7\pi}{6}$ $\quad \frac{11\pi}{6}$

$4\cos\theta - 1 = \cos\theta$

$4\cos\theta = 1 + \cos\theta$

$3\cos\theta = 1$

$\cos\theta = \frac{1}{3}$



$\cos^{-1}\left(\frac{1}{3}\right) = \theta = 1.23$

$2\pi - 1.23 = 5.05$

Solve each equation for $0 \le \theta \le 2\pi$.

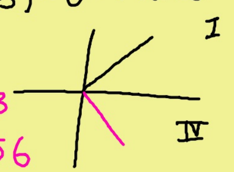
$7\cos\theta - 3 = 2\cos\theta$

$7\cos\theta = 3 + 2\cos\theta$

$5\cos\theta = 3$

$\cos\theta = \frac{3}{5}$

$\cos^{-1}\left(\frac{3}{5}\right) = \theta = .9273$



$2\pi - .9273$

5.356

$\sin\theta\cos\theta - \cos\theta = 0$

$\cos\theta(\sin\theta - 1) = 0$

zero-product property

$\cos\theta = 0$ $\quad \sin\theta - 1 = 0$

$\frac{\pi}{2}, \frac{3\pi}{2}$ $\quad \sin\theta = 1$

$\frac{\pi}{2}$

Solve each equation for $0 \le \theta \le 2\pi$.

$2\sin\theta\cos\theta - \cos\theta = 0$

$\cos\theta(2\sin\theta - 1) = 0$

$\cos\theta = 0$ $\quad 2\sin\theta - 1 = 0$

$\frac{\pi}{2}, \frac{3\pi}{2}$ $\quad 2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\frac{\pi}{6}, \frac{5\pi}{6}$

$2\cos^2\theta - 1 = 0$

$2\cos^2\theta = 1$

$\sqrt{\cos^2\theta} = \pm\sqrt{\frac{1}{2}}$

$\cos\theta = \pm\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\cos\theta = +\frac{\sqrt{2}}{2} \quad -\frac{\sqrt{2}}{2}$

$\frac{\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$

homework:
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