2.3 Direct Variation

Direct variation is a linear function of the form y = kx where k ≠ 0

(This is just a line which passes through the origin and k is your slope)

Constant of variation (k) is the ratio $k = \frac{y}{x}$

To vary directly, it must be linear and pass through the origin.

For each table, determine whether y varies directly with x. If so, find the constant of variation and write the equation.

x	y
-6	-2
3	1
12	4

y/x

$\frac{-2}{-6} = \frac{1}{3}$

$\frac{1}{3}$

$\frac{4}{12} = \frac{1}{3}$

yes

$k = \frac{1}{3}$

$y = \frac{1}{3}x$

x	y
-1	-2
3	4
6	7

$\frac{y}{x}$

$\frac{-2}{-1} = 2$

$\frac{4}{3} \neq 2$

not direct variation

For each equation, determine whether y varies directly with x. If so, find the constant of variation and write the equation.

$y = \frac{x}{2}$

yes, direct variation

$k = \frac{1}{2}$

$2y - 1 = x$

$+1 \quad +1$

$\frac{2}{2}y = \frac{x}{2} + \frac{1}{2}$

$y = \frac{x}{2} + \frac{1}{2}$

not direct variation

A dripping faucet wastes a cup of water (y) if it drips for three minutes (x).

The amount of water varies directly with the time the faucet drips.

linear, constant = 0

A) find k and write an equation. $k = \frac{y}{x} = \frac{1}{3}$ $y = \frac{1}{3}x$

B) for how long must the faucet drip to waste 4.5 cups of water? (y)

$3 \cdot 4.5 = \frac{1}{3}x \cdot 3$

$13.5 \text{ min} = x$

Suppose y varies directly with x. Find the missing value for the direct variation.

linear, constant = 0

If y = 4 when x = 3, find y when x = 6.

$k = \frac{y}{x}$, set up a proportion

$\frac{4}{3} = \frac{y}{6}$ $\quad 24 = 3y$ $\quad 8 = y$

If y = 10 when x = -3, find x when y = 2.

$\frac{10}{-3} = \frac{2}{x}$ $\quad \frac{10x}{10} = \frac{-6}{10}$

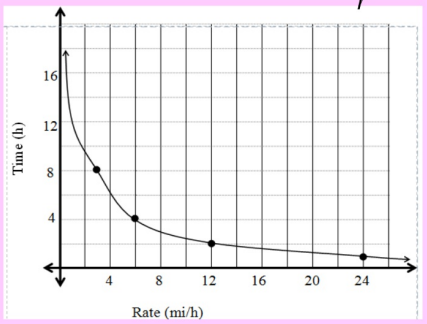
$x = -\frac{3}{5}$

9.1 Inverse Variation

Inverse Variation is a function of the form $y = \frac{k}{x}$ or xy = k where k≠0

The table and graph show the time needed to bike 24 miles if you pedal at different rates. Notice that as the rate increases the time decreases. Doubling the rate halves the time. The inverse variation $t = \frac{24}{r}$ models the situation.

rate (mi/hr)	time (hr)
3	8
6	4
12	2
24	1



Suppose x and y vary inversely. Write a function that models inverse variation.

divide by x

x = 7 when y = 4

$y = \frac{k}{x} \Rightarrow xy = k$

$4(7) = 28$

$y = \frac{28}{x}$

x = 0.7 when y = 1.4

$k = 0.7(1.4)$

$k = .98$

$y = \frac{.98}{x}$

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Is the relationship between the values in each table a direct variation, an inverse variation, or neither? Write functions to model the direct and inverse variations.

x	y	$\frac{y}{x}$
0.8	0.9	$\frac{0.9}{0.8} = \frac{9}{8}$
0.6	1.2	$\frac{1.2}{0.6} = 2$
0.4	1.8	

xy
0.8(0.9)=.72
0.6(1.2)=.72
0.4(1.8).72

x	y	$\frac{y}{x}$
2	3.2	$\frac{3.2}{2} = 1.6$
4	1.6	$\frac{1.6}{4} \neq 1.6$
6	1.1	

xy
2(3.2)=6.4
4(1.6)=6.4
6(1.1)=6.6

inverse varation

$y = \frac{.72}{x}$

neither

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Is the relationship between the values in each table a direct variation, an inverse variation, or neither? Write functions to model the direct and inverse variations.

x	y	$\frac{y}{x}$
1.2	18	$\frac{18}{1.2} = 15$
1.4	21	$\frac{21}{1.4} = 15$
1.6	24	$\frac{24}{1.6} = 15$

$y = 15x$

direct variation

x	y	$\frac{y}{x}$
2	0.7	$\frac{0.7}{2} = 0.35$
4	0.35	$\frac{0.35}{4} = .175$
14	0.1	$\frac{0.1}{14}$

xy
2(.7)=1.4
4(.35)=1.4
14(.1)=1.4

inverse variation

$y = \frac{1.4}{x}$

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Heart rates and life spans of most mammals are inversely proportional. Use the data to write a function that modesl this inverse variation. Use your function to answer the questions.

Heart Rate and Life Span		
mammal	heart rate (beats/min)	life span (min)
mouse	634	1,576,800
rabbit	158	6,307,200
lion	76	13,140,000
horse	63	15,786,000

xy
999,691,200
996,537,600
998,640,000
993,384,000

$k \approx 1{,}000{,}000{,}000$

$y = \frac{1{,}000{,}000{,}000}{x}$

$y = \frac{1{,}000{,}000{,}000}{190}$

$y = 5{,}263{,}158$ min

$36{,}792{,}000 = \frac{1 \text{ bil}}{x}$

$x = 27$ beats/min

(A) a squirrel's heart rate is 190 bpm. Estimate its life span.

(B) an elephant's life span is 70 years. Estimate its average heart rate.

70(365)(24)(60) = 36,792,000 minutes

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homework:

page 74 # 4-28 x 4 direct variation $y = kx$

page 481 # 1-15, 37, 38, 43, 44

inverse variation

$$y = \frac{k}{x}$$