4.1-4.3 Matrix Addition, Subtraction and Multiplication

A Matrix is a rectangular array of numbers written within brackets. You represent a matrix with a capital letter and classify it by its dimensions. The number of horizontal rows and vertical columns determine the dimensions of a matrix.

Write the dimensions of each matrix:

$A\begin{bmatrix} 4 & 6 & 0 \\ -2 & 0.5 & 17 \end{bmatrix}$ 2 × 3

$B\begin{bmatrix} 8 & -3 & 15 \end{bmatrix}$ 1 × 3

$Z\begin{bmatrix} 10 & 0 \\ 1 & -5 \\ -6.4 & 9 \end{bmatrix}$ 3 × 2

Each number in a matrix is a matrix element. You can identify a matrix element by its position within the matrix. Use a lowercase letter with subscripts. The subscripts represent the element's row number and column number.

Identify each matrix element:

$A\begin{bmatrix} 4 & 6 & 0 \\ -2 & 0.5 & 17 \end{bmatrix}$

a_{12} 6

a_{23} 17

a_{22} 0.5

a_{rc}

You can perform matrix addition/subtraction on matrices with equal dimensions by adding/subtracting the corresponding elements (elements in the same position in each matrix).

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} + \begin{bmatrix} r & s & t \\ u & v & w \end{bmatrix} = \begin{bmatrix} a+r & b+s & c+t \\ d+u & e+v & f+w \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} - \begin{bmatrix} r & s & t \\ u & v & w \end{bmatrix} = \begin{bmatrix} a-r & b-s & c-t \\ d-u & e-v & f-w \end{bmatrix}$$

Solve:

$$\begin{bmatrix} 6 & -9 & 7 \\ -2 & 1 & 8 \end{bmatrix} + \begin{bmatrix} -4 & 3 & 0 \\ 6 & 5 & 10 \end{bmatrix} \qquad \begin{bmatrix} 6 & -9 & 7 \\ -2 & 1 & 8 \end{bmatrix} + \begin{bmatrix} +4 & -3 & -0 \\ -6 & -5 & -10 \end{bmatrix}$$

$$\begin{bmatrix} 6+(-4) & -9+3 & 7+0 \\ -2+6 & 1+5 & 8+10 \end{bmatrix} \qquad \begin{bmatrix} 10 & -12 & 7 \\ -8 & -4 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -6 & 7 \\ 4 & 6 & 18 \end{bmatrix}$$

Solving Matrix Equations:

A matrix equation is an equation in which the variable is a matrix. you can use the addition and subtraction properties of equality to solve.

$$X - \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 8 & 9 \end{bmatrix}$$

$$+\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 2 \\ 11 & 11 \end{bmatrix}$$

$$X + \begin{bmatrix} -1 & 0 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ -4 & 4 \end{bmatrix}$$

$$- \begin{bmatrix} -1 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} +1 & -0 \\ -2 & -5 \end{bmatrix}$$

$$X = \begin{bmatrix} 11 & 7 \\ -6 & -1 \end{bmatrix}$$

Solve for x, y and z.

$$\begin{bmatrix} x+8 & -5 \\ 3 & -y \end{bmatrix} = \begin{bmatrix} 38 & -5 \\ 3 & 4y-10 \end{bmatrix}$$

$$x + 8 = 38$$

$$x = 30$$

$$-y = 4y - 10$$

$$-4y \quad -4y$$

$$-5y = -10$$

$$y = 2$$

Solve for x, y and z.

$$\begin{bmatrix} 2x-y & -3 \\ 8 & -4x+2y \end{bmatrix} = \begin{bmatrix} 15 & x+y \\ z & -30 \end{bmatrix}$$

$z = 8$

$x = 4$

$y = -7$

$$+ \quad \begin{array}{r} 2x - y = 15 \\ x + y = -3 \end{array}$$

$$3x = 12$$

$$x = 4$$

$$4 + y = -3$$

$$y = -7$$

Equal matrices are matrices with the same dimensions and equal corresponding elements.

Are the following matrices equal?

$$\begin{bmatrix} -1 & \frac{1}{2} & 2.5 \\ 0 & \frac{3}{4} & 4 \end{bmatrix} \quad \begin{bmatrix} 4-5 & 0.5 & 8-4.5 \\ 3-3 & -0.75 & 1+3 \end{bmatrix}$$

$$\begin{bmatrix} -1 & \frac{1}{2} & 3.5 \\ 0 & -\frac{3}{4} & 4 \end{bmatrix}$$

not equal

$$[2 \quad 4 \quad 6] \begin{bmatrix} \frac{4}{2} \\ \frac{12}{3} \\ \frac{24}{4} \end{bmatrix}$$

$(1 \times 3) \neq (3 \times 1)$

not equal

You can multiply a matrix by a real number called a scalar. $3\begin{bmatrix}3 & 5\\ 2 & 8\end{bmatrix} = \begin{bmatrix}9 & 15\\ 6 & 24\end{bmatrix}$

Use matrices A and B to find each sum or difference. $A = \begin{bmatrix}2 & 3 & -7\\ 1 & 4 & 5\end{bmatrix}$ $B = \begin{bmatrix}3 & 0 & 6\\ -1 & 8 & 2\end{bmatrix}$

5B−4A

$\begin{bmatrix}15 & 0 & 30\\ -5 & 40 & 10\end{bmatrix} + \begin{bmatrix}-8 & -12 & +28\\ -4 & -16 & -20\end{bmatrix}$

$\begin{bmatrix}7 & -12 & 58\\ -9 & 24 & -10\end{bmatrix}$

A+6B

$\begin{bmatrix}2 & 3 & -7\\ 1 & 4 & 5\end{bmatrix} + \begin{bmatrix}18 & 0 & 36\\ -6 & 48 & 12\end{bmatrix}$

$\begin{bmatrix}20 & 3 & 29\\ -5 & 52 & 17\end{bmatrix}$

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Use the properties of scalar multiplication to solve matrix equations.

$4X + 2\begin{bmatrix}3 & 4\\ -2 & 1\end{bmatrix} = \begin{bmatrix}10 & 0\\ 4 & 2\end{bmatrix}$

$4X + \begin{bmatrix}6 & 8\\ -4 & 2\end{bmatrix} = \begin{bmatrix}10 & 0\\ 4 & 2\end{bmatrix}$

$-\begin{bmatrix}6 & 8\\ -4 & 2\end{bmatrix} + \begin{bmatrix}-6 & -8\\ +4 & -2\end{bmatrix}$

$\frac{1}{4}\cdot 4X = \frac{1}{4}\begin{bmatrix}4 & -8\\ 8 & 0\end{bmatrix}$

$X = \begin{bmatrix}1 & -2\\ 2 & 0\end{bmatrix}$

$-3Y + 2\begin{bmatrix}6 & 9\\ -12 & 15\end{bmatrix} = \begin{bmatrix}27 & -18\\ 30 & 6\end{bmatrix}$

$-3Y + \begin{bmatrix}12 & 18\\ -24 & 30\end{bmatrix} = \begin{bmatrix}27 & -18\\ 30 & 6\end{bmatrix}$

$-\begin{bmatrix}12 & 18\\ -24 & 30\end{bmatrix} + \begin{bmatrix}-12 & -18\\ +24 & -30\end{bmatrix}$

$-3Y = \begin{bmatrix}15 & -36\\ 54 & -24\end{bmatrix}$

$\cdot -\frac{1}{3}$ $\cdot -\frac{1}{3}$

$Y = \begin{bmatrix}-5 & 12\\ -18 & 8\end{bmatrix}$

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The product of two matrices A and B exists only if the number of columns of A is equal to the number of rows of B.

Use matrices G and H to determine whether the products GH and HG are defined (exist) or undefined (do not exist).

$G\begin{bmatrix}2 & 3\\ -1 & 8\\ 4 & 0\end{bmatrix}$ $H\begin{bmatrix}8 & 0\\ 2 & -5\end{bmatrix}$

GH

$(3\times\underline{2})(\underline{2}\times 2)$

same

defined

HG

$(2\times\underline{2})(\underline{3}\times 2)$

different

undefined

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To perform matrix multiplication, multiply the elements of each row in the first matrix by the elements of each column in the second matrix. Add the products.

Multiply. $\begin{bmatrix}-1 & 0\\ 3 & -4\end{bmatrix} \bullet \begin{bmatrix}-3 & 3\\ 5 & 0\end{bmatrix}$

$= \begin{bmatrix}(-1)(-3)+0\cdot 5 = 3+0 = 3 & \\ & \end{bmatrix}$

$= \begin{bmatrix}3 & (-1)(3)+0\cdot 0 = -3+0 \\ & \end{bmatrix}$

$= \begin{bmatrix}3 & -3\\ (3)(-3)+(-4)(5) = -9-20 & \end{bmatrix}$

$= \begin{bmatrix}3 & -3\\ -29 & 3(3)+-4(0) = 9+0\end{bmatrix}$

$= \begin{bmatrix}3 & -3\\ -29 & 9\end{bmatrix}$

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Multiply. $\begin{bmatrix} 8 & 6 & 13 \end{bmatrix} \begin{bmatrix} 9 \\ 30 \\ 20 \end{bmatrix}$

$(1 \times 3)(3 \times 1)$

$8 \cdot 9 + 6 \cdot 30 + 13 \cdot 20$

$72 + 180 + 260 = [512]$

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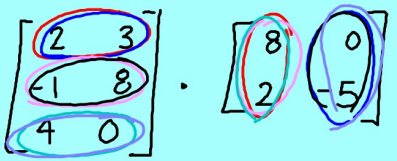
Multiply GH and/or HG from above.



$2 \cdot 8 + 3 \cdot 2 = 22$

$-1 \cdot 8 + 8 \cdot 2 = 8$

$4 \cdot 8 + 0 \cdot 2 = 32$

$\begin{bmatrix} -15 \\ -40 \\ 0 \end{bmatrix} = \begin{matrix} 2 \cdot 0 + 3(-5) \\ -1 \cdot 0 + 8(-5) \\ 4 \cdot 0 + 0(-5) \end{matrix}$

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Homework:

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page 174 # 2-10 even, 16, 17, 24

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