4.5 and 4.6 Determinants and Inverses

$6 \cdot \_\_1\_\_ = 6$ $\quad 6 \cdot \_\frac{1}{6}\_ = 1$

A square matrix is a matrix with the same number of columns and rows.

Multiplicative Identity Matrix: For any n x n matrix with 1s along the main diagonal and 0s elsewhere. $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Multiplicative Inverse: If A and X are n x n, and AX = XA = I, then X is the multiplicative inverse of A (written A^{-1}). $AA^{-1} = A^{-1}A = I$.

Show that A and B are multiplicative inverses of each other.

$$A = \begin{bmatrix} 3 & -1 \\ 7 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0.1 & 0.1 \\ -0.7 & 0.3 \end{bmatrix}$$

AB = I

$$\begin{bmatrix} 3 & -1 \\ 7 & 1 \end{bmatrix}\begin{bmatrix} .1 & .1 \\ -.7 & .3 \end{bmatrix} = \begin{bmatrix} 3(.1) + .3 & 3(.1) = .3 \\ -1(-.7) = .7 & -1(.3) = -.3 \\ \hline 1 & 0 \\ \\ 7(.1) = .7 & 7(.1) = .7 \\ 1(-.7) = -.7 & +1(.3) = .3 \\ \hline 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -1 \\ 7 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0.1 & 0.1 \\ -0.7 & 0.3 \end{bmatrix}$$

BA = I

$$\begin{bmatrix} .1 & .1 \\ -.7 & .3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 7 & 1 \end{bmatrix} = \begin{bmatrix} .1(3) = .3 & .1(-1) = -.1 \\ .1(7) = .7 & .1(1) = .1 \\ \hline 1 & 0 \\ \\ -.7(3) = -2.1 & -.7(-1) = .7 \\ .3(7) = 2.1 & .3(1) = .3 \\ \hline 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Every square matrix with real number elements has a real number determinant. Determinants help you find inverses.

Determinant for a 2 x 2 matrix $A\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$

Symbols for the determinant det A $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$.

Evaluate the determinant.

$\det\begin{bmatrix} 7 & 8 \\ -5 & -9 \end{bmatrix}$

$-63-(-40)$

$-63+40$

-23

$\det\begin{bmatrix} 4 & -3 \\ 5 & 6 \end{bmatrix}$

$24-(-15)$

$24+15$

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Inverse of a 2 x 2 Matrix. If the determinant of a matrix is 0, then there is no inverse matrix.

If $A\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Find the inverse of $Y = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

1. get determinant $6-4=2$

2. $\frac{1}{2}\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3/2 & -2 \\ -1/2 & 1 \end{bmatrix}$

Find X. $\begin{bmatrix} 9 & 25 \\ 4 & 11 \end{bmatrix} X = \begin{bmatrix} 3 \\ -7 \end{bmatrix}$

inverse

$\det \quad 99-100=-1$

$\frac{1}{-1}\begin{bmatrix} 11 & -25 \\ -4 & 9 \end{bmatrix} = \begin{bmatrix} -11 & 25 \\ 4 & -9 \end{bmatrix}$

$\begin{bmatrix} -11 & 25 \\ 4 & -9 \end{bmatrix}\begin{bmatrix} 9 & 25 \\ 4 & 11 \end{bmatrix} X = \begin{bmatrix} -11 & 25 \\ 4 & -9 \end{bmatrix}\begin{bmatrix} 3 \\ -7 \end{bmatrix} = \begin{bmatrix} -11(3) & -33 \\ +25(-7) & -175 \\ & -208 \\ 4(3) & 12 \\ -9(-7) & 63 \\ & 75 \end{bmatrix}$

$X = \begin{bmatrix} -208 \\ 75 \end{bmatrix}$

Find X. $\begin{bmatrix} -2 & -5 \\ 1 & 3 \end{bmatrix} X = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

$\det \quad (-6)-(-5) = -6+5 = -1$

inverse $\frac{1}{-1}\begin{bmatrix} 3 & 5 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -3 & -5 \\ 1 & 2 \end{bmatrix}$

$\begin{bmatrix} -3 & -5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -2 & -5 \\ 1 & 3 \end{bmatrix} X = \begin{bmatrix} -3 & -5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 2 \end{bmatrix}$

$\begin{bmatrix} -3(-2)=6 \\ -5(2)=-10 \\ \quad -4 \\ 1(-2)=-2 \\ 2(2)=4 \\ \quad 2 \end{bmatrix}$

$X = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$

Find the determinant.

$$A = \begin{bmatrix} 8 & -4 & 3 \\ -2 & 9 & 5 \\ 1 & 6 & 0 \end{bmatrix}$$

8 −4 3 8 −4
−2 9 5 −2 9
1 6 0 1 6

$(0 + (-20) + (-36)) - (27 + 240 + 0)$

$-56 - 267 = -323$

homework

page 199 # 2, 4, 8, 14, 16, 22, 42 (8 & 22 by hand)

page 205 # 2, 6, 8, 10, 11, 29 (2 by hand)