4.7 Inverse Matrices and Systems

You can represent a system of equations with a matrix equation.

System of equations:

$$x + 2y = 5$$
$$3x + 5y = 14$$

Matrix equation:

$$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 14 \end{bmatrix}$$

Each matrix in an equation of the form AX=B has a name.

Coefficient Matrix (A) $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

Variable Matrix (X) $\begin{bmatrix} x \\ y \end{bmatrix}$

Constant Matrix (B) $\begin{bmatrix} 5 \\ 14 \end{bmatrix}$

Write the system as a matrix equation. Identify the coefficient matrix, the variable matrix, and the constant matrix.

$$-b + c = 4$$
$$a + b - c = 0$$
$$2a + 3c = 11$$

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 11 \end{bmatrix}$$

coefficient matrix

Variable matrix

Solve the system. $2x + 3y = 11$, $x + 2y = 6$

$$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ 6 \end{bmatrix}$$

det $4 - 3 = 1$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 11 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \quad (4,1)$$

Solve the system. $x + 2y = 5$, $3x + 5y = 14$

$$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 14 \end{bmatrix}$$

det $5 - 6 = -1$

$$\frac{1}{-1} \begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad (3,1)$$

Solve the system. $\begin{aligned} -b + c &= 4 \\ a + b - c &= 0 \\ 2a + 3c &= 11 \end{aligned}$

$$\underbrace{\begin{bmatrix} 0 & -1 & 1 \\ 1 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}}_{[A]} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \underbrace{\begin{bmatrix} 4 \\ 0 \\ 11 \end{bmatrix}}_{[B]}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = A^{-1}B \qquad \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix} \quad (4, -3, 1)$$

Page 5

Solve the system. $\begin{aligned} -3x - 4y + 5z &= 11 \\ -2x + 7y \quad &= -6 \\ -5x + y - z &= 20 \end{aligned}$

$$\begin{bmatrix} -3 & -4 & 5 \\ -2 & 7 & 0 \\ -5 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -6 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}B \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4.0 \\ -2.0 \\ -1.8 \end{bmatrix}$$

Page 6

Solve the system. $\begin{aligned} 7x + 3y + 2z &= 13 \\ -2x + y - 8z &= 26 \\ x - 4y + 10z &= -13 \end{aligned}$

$$\begin{bmatrix} 7 & 3 & 2 \\ -2 & 1 & -8 \\ 1 & -4 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 26 \\ -13 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -12 \\ -7 \end{bmatrix} \qquad (9, -12, -7)$$

Page 7

Margo bought 3 sheets and 5 towels for \$137.50. Sue bought 4 sheets and 2 towels for \$118.00. How much did one sheet and one towel cost?

x = sheets
y = towels

$$3x + 5y = 137.50$$
$$4x + 2y = 118$$

\$22.50 sheet
\$14 towel

$$\begin{bmatrix} 3 & 5 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 137.5 \\ 118 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}B$$

Page 8

Jane bought 2 large beads and 3 small beads for \$5.55. Mae bought 3 large beads and 4 small beads for \$8.05. What is the price for each type of bead?

x = large
y = small

$2x + 3y = 5.55$
$3x + 4y = 8.05$

large \$1.95
small \$0.55

$$\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5.55 \\ 8.05 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1.95 \\ 0.55 \end{bmatrix}$$

When the coefficient matrix of a system has an inverse, the system has a unique solution. When the coefficient matrix does not have an inverse (because the determinant = 0), the system does NOT have a unique solution. In that case, the system either has no solution or has an infinite number of solutions.

What happens when you try to use the inverse matrix to solve a system of equations (graphing calculator) and the system does not have a unique solution? ERR:SINGULAR MATRIX

$2(2x + 3y = 6)$
$4x + 6y = 12$

$4x + 6y = 12$
$4x + 6y = 12$

$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$$

$12 - 12 = 0$

same line
infinite solutions
dependent

$2(2x + 3y = 6)$
$4x + 6y = 10$

$4x + 6y = 12$
$4x + 6y = 10$

parallel lines
no solution
inconsistent

homework
page 213 # 4, 6, 14, 30, 33, 35, 42, 49
(4 use graphing calc, 6, 49 do by hand)