5.7 Completing the Square

Review the steps for Completing the Square:

1. Make sure $a = 1$
2. Set equation to $ax^2 + bx = c$
3. Find $(b/2)^2$
4. Add $(b/2)^2$ to both sides of the equation
5. Complete the square
6. Take the square root of both sides
7. Solve both mini equations.

Solve each quadratic equation by Completing the Square

$x^2 - 12x + 5 = 0$

$\left(\frac{-12}{2}\right)^2 = (-6)^2 = 36$

$x^2 - 12x + 36 = -5 + 36$

$\sqrt{(x-6)^2} = \sqrt{31}$

$x - 6 = \pm\sqrt{31}$

$+6 \quad +6$

$x = 6 \pm \sqrt{31}$

Solve each quadratic equation by Completing the Square

$x^2 + 4x - 4 = 0$

$x^2 + 4x + 4 = 4 + 4$

$\left(\frac{4}{2}\right)^2 = 2^2 = 4$

$\sqrt{(x+2)^2} = \sqrt{8}$

$x + 2 = \pm 2\sqrt{2}$

$-2 \quad -2$

$x = -2 \pm 2\sqrt{2}$

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$2 \cdot 4$

$2 \cdot 2$

$2\sqrt{2}$

The method of completing the square can only be used when the coefficient of the x^2 term is 1. If the x^2 term is a different coefficient, divide every term of the equation by that coefficient to obtain the desired form. Solve each quadratic equation by Completing the Square

$5x^2 = 6x + 8$

$5x^2 - 6x = 8$

$x^2 - \frac{6}{5}x + \frac{9}{25} = \frac{8}{5} + \frac{9}{25}$

$\left(\frac{-\frac{6}{5}}{2}\right)^2 = \left(-\frac{6}{5} \cdot \frac{1}{2}\right)^2 = \left(-\frac{3}{5}\right)^2 = \frac{9}{25}$

$\sqrt{\left(x - \frac{3}{5}\right)^2} = \sqrt{\frac{49}{25}}$

$x - \frac{3}{5} = \pm\frac{7}{5}$

$+\frac{3}{5} \quad +\frac{3}{5}$

$\frac{7}{5} + \frac{3}{5} = \frac{10}{5} = 2$

$-\frac{7}{5} + \frac{3}{5} = \frac{-4}{5}$

$\frac{8}{5} = \frac{40}{25}$

$x = 2, \frac{-4}{5}$

Solve each quadratic equation by Completing the Square

$\frac{2x^2}{2} + \frac{x}{2} = \frac{6}{2}$

$x^2 + \frac{1}{2}x + \frac{1}{16} = 3 + \frac{1}{16}$

$\left(\frac{\frac{1}{2}}{2}\right)^2 = \left(\frac{1}{2}\cdot\frac{1}{2}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

$\sqrt{\left(x + \frac{1}{4}\right)^2} = \pm\sqrt{\frac{49}{16}}$

$x + \frac{1}{4} = \pm\frac{7}{4}$

$-\frac{1}{4} \quad -\frac{1}{4}$

$x = \frac{3}{2}, -2$

$\frac{3}{1} = \frac{48}{16} + \frac{1}{16}$

$\frac{7}{4} - \frac{1}{4} = \frac{6}{4} = \frac{3}{2}$

$-\frac{7}{4} - \frac{1}{4} = \frac{-8}{4} = -2$

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Solve each quadratic equation by Completing the Square

$2x^2 - 5x + 1 = 0$ $= x^2 - \frac{5}{2}x + \frac{25}{16} = -\frac{1}{2} + \frac{25}{16}$

$\left(\frac{-\frac{5}{2}}{2}\right)^2 = \left(\frac{-5}{4}\right)^2 = \frac{25}{16}$

$\sqrt{\left(x - \frac{5}{4}\right)^2} = \pm\sqrt{\frac{17}{16}}$

$-\frac{1}{2} = \frac{-8}{16} + \frac{25}{16} = \frac{17}{16}$

$x - \frac{5}{4} = \pm\frac{\sqrt{17}}{4}$

$+\frac{5}{4} \quad +\frac{5}{4}$

$x = \frac{5 \pm \sqrt{17}}{4}$

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Solve each quadratic equation by Completing the Square

$6x - 3x^2 = -12$

$\frac{-3x^2}{-3} + \frac{6x}{-3} = \frac{-12}{-3}$

$\left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$

$x^2 - 2x + 1 = 4 + 1$

$\sqrt{(x-1)^2} = \sqrt{5}$

$x - 1 = \pm\sqrt{5}$

$+1 \quad +1$

$x = 1 \pm \sqrt{5}$

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Rewriting a Function in Vertex form by completing the square.

Vertex form of a quadratic equation is $y = a(x-h)^2 + k$

When completing the square, you can work with just one side of the function equation. For example, $y = x^2 + 6x + 2$ is equivalent to $y = x^2 + 6x \underline{+9} + 2 \underline{-9}$. When you add 9 to the right side of the equation you must remember to subtract 9 from the same side to keep the equation equivalent (additive inverses).

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Example

$y = x^2 + 6x + 2$ Find $(b/2)^2 = (6/2)^2 = 3^2$

$y = x^2 + 6x + \underline{3^2} + 2 \underline{- 3^2}$ Complete the square, add & subtract 3^2 on the right.

$y = (x + 3)^2 + 2 - 9$ Factor the perfect square trinomial

$y = (x + 3)^2 - 7$ Simplify & leave in vertex form.

Write each equation in vertex form

$\left(\frac{5}{2}\right)^2 = \frac{25}{4}$

$\left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$

$y = x^2 - 10x - 2$

$y = x^2 - 10x + \underline{25} - 2 - \underline{25}$

$y = (x - 5)^2 - 27$

$y = x^2 + 5x + 3$

$y = x^2 + 5x + \underline{\frac{25}{4}} + 3 - \underline{\frac{25}{4}}$

$y = (x + \frac{5}{2})^2 - \frac{13}{4}$

$\frac{3}{1} = \frac{12}{4} - \frac{25}{4}$

$-\frac{13}{4}$

When the coefficient of the quadratic term is not 1 ($a \neq 1$), factor out the coefficient from the quadratic and linear terms. Remember that the factored coefficient is distributed to all the terms within the parentheses.

Example: $y = -x^2 - 2x + 3$

$y = -1(x^2 + 2x + \_\_\_\_\_) + 3 + (-1)(- \_\_\_\_\_)$ Add $(b/2)^2$ inside parentheses & Distribute and add opposite outside

$y = -(x^2 + 2x + 1) + 3 + 1$

$y = -(x + 1)^2 + 4$

Write each equation in vertex form.

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$\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

$y = (2x^2 - 8x) + 1$

$y = 2(x^2 - 4x + \underline{4}) + 1 - \underline{4}(2)$

$y = 2(x - 2)^2 + 1 - 8$

$y = 2(x - 2)^2 - 7$

PQT

$y = (-x^2 + 4x) - 1$

$y = -(x^2 - 4x + \underline{4}) - 1 - \underline{4}(-1)$

$y = -(x - 2)^2 - 1 + 4$

$y = -(x - 2)^2 + 3$