5.1 and 5.2 Properties of Parabolas

The vertex form of a quadratic function is $y = a(x-h)^2 + k$

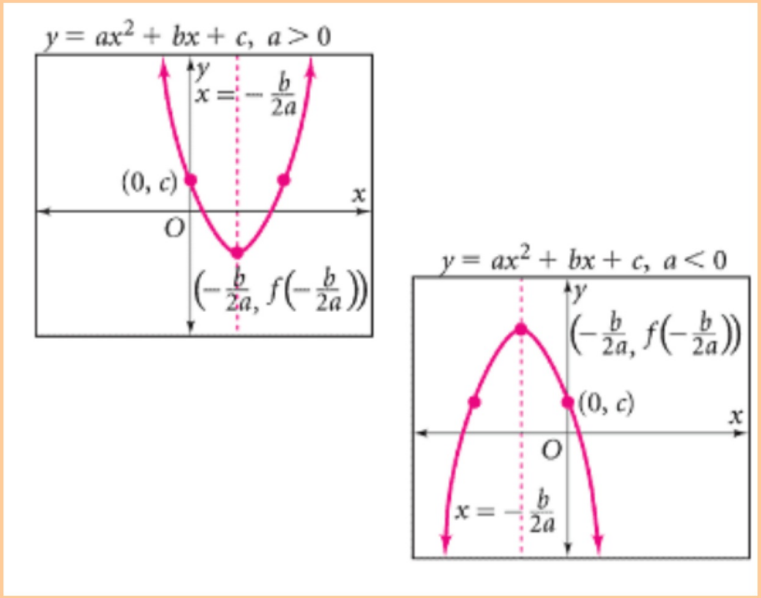
The standard form of a quadratic function is $y = ax^2 + bx + c$

When b = 0, the function simplifies to

The graph of $y = ax^2 + c$ is a parabola with an axis of symmetry at x = 0, the y-axis. The vertex of the graph is the minimum, (0, c).

In standard form,

When a > 0, the parabola opens up and the vertex is a minimum, When a<0, the parabola opens down and the vertex is a maximum. The x-coordinate of the vertex is $\frac{-b}{2a}$ and you find the y-coordinate of the vertex by substituting the x-coordinate into the equation. The axis of symmetry is the vertical line through the vertex. The y-intercept is the constant.



Graph the functions. Label the vertex and axis of symmetry. Find the maximum or minimum.

$y = \frac{1}{3}x^2 + 1$

x = 0

(0, 1)

$y = x^2 - 2x - 3$

$x = \frac{-b}{2a} = \frac{2}{2 \cdot 1} = \frac{2}{2} = 1$

$y = 1^2 - 2 \cdot 1 - 3 = 1 - 2 - 3 = -4$

x = 1

(1, -4)

$y = -2x^2 + 4x + 2$

$x = \frac{-b}{2a} = \frac{-4}{2(-2)} = \frac{-4}{-4} = 1$

$y = -2(1)^2 + 4(1) + 2$

$y = -2 + 4 + 2 = 4$

x = 1

(1, 4)

Graph the functions. Label the vertex and axis of symmetry. Find the maximum or minimum.

$y = \frac{1}{2}x^2 + x + 3$

$x = \frac{-b}{2a} = \frac{-1}{2(\frac{1}{2})} = -\frac{1}{1} = -1$

$y = \frac{1}{2}(-1)^2 + (-1) + 3$

$= \frac{1}{2} - 1 + 3$

$= 2\frac{1}{2}$

(-1, 2½) - Min

x = -1

The number of weekend get-away packages a hotel can sell is modeled by $-0.12p + 60$, where p is the price of a get-away package. The income is the product of the price and the number of packages sold. What price will maximize the income? What is the maximum income?

$$I = p(-0.12p + 60)$$

$$-0.12p^2 + 60p$$

$$p = \frac{-60}{2(-0.12)} = \frac{-60}{-.24} = \$250 \text{ max price}$$

$$I = -0.12(250)^2 + 60(250) = \$7500$$

Find a quadratic function to model the values in the tables by using a matrix.

x	y
2	3
3	13
4	29

$y = ax^2 + bx + c$

$3 = a(2)^2 + b(2) + c$

$3 = 4a + 2b + c$

$13 = a(3)^2 + b(3) + c$

$13 = 9a + 3b + c$

$29 = a(4)^2 + b(4) + c$

$29 = 16a + 4b + c$

$$\underset{A}{\begin{bmatrix} 4 & 2 & 1 \\ 9 & 3 & 1 \\ 16 & 4 & 1 \end{bmatrix}} \underset{X}{\begin{bmatrix} a \\ b \\ c \end{bmatrix}} = \underset{C}{\begin{bmatrix} 3 \\ 13 \\ 29 \end{bmatrix}}$$

$X = A^{-1}B$ (use graphing calculator to do your matrices)

$a = 3 \quad b = -5 \quad c = 1$

$y = 3x^2 - 5x + 1$

Find a quadratic function to model the values in the tables by using a matrix.

x	y
-2	-17
1	10
5	-10

$-17 = a(-2)^2 + b(-2) + c$

$-17 = 4a - 2b + c$

$10 = a \cdot 1^2 + b \cdot 1 + c$

$10 = a + b + c$

$-10 = a \cdot 5^2 + b \cdot 5 + c$

$-10 = 25a + 5b + c$

$$\underset{A}{\begin{bmatrix} 4 & -2 & 1 \\ 1 & 1 & 1 \\ 25 & 5 & 1 \end{bmatrix}} \underset{X}{\begin{bmatrix} a \\ b \\ c \end{bmatrix}} = \underset{B}{\begin{bmatrix} -17 \\ 10 \\ -10 \end{bmatrix}}$$

$X = A^{-1}B$

$a = -2 \quad b = 7 \quad c = 5$

$y = -2x^2 + 7x + 5$

Use the graphing calculator to make a scatter plot and then use QuadReg to find the quadratic function. The table shows data about a wavelength x (in meters) and the wave speed y (in meters per second) of deep water ocean waves. Model the data with a quadratic function. Graph the data and the function. Use the model to estimate the wave speed of a deep water wave that has a wavelength of 6 meters.

wavelength x (m)	wave speed y (m/s)
3	6
5	16
7	31
8	40

$y = .59x^2 + .34x - .33$

put in Y= in calculator, check table

$x = 6 \quad y \approx 29$ m/s

homework:

page 237 # 3-12 x 3, 20 (use a matrix) 21, 22

page 244 # 3-39 x 3