5.6 Complex Numbers

When working with quadratics, to solve for x we often use square roots. $x^2 = 100$ $\quad x^2 - 8 = 0$

Perfect Squares need to be memorized.

1 4 9 16 25 36 49 64 81 100 121 144 169 196 225

Sometimes we have roots that are negative. $x^2 = -4$ $\quad x^2 + 8 = 0$

Negative Square Roots are Complex Numbers because they are not located on the Real Number Line.

The Complex Number $i = \sqrt{-1}$

Square Root of a negative number: for any positive real number, a,

$$\sqrt{-a} = i\sqrt{a}$$

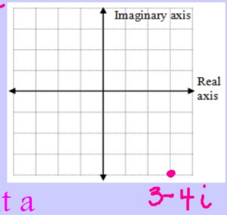
Page 1

Simplify $\quad \sqrt{-2} \qquad \sqrt{-12} \qquad \sqrt{-36}$

$\sqrt{-1 \cdot 2}$ $\quad \sqrt{-1 \cdot 12}$ $\quad 12$ $\quad \sqrt{-1 \cdot 36}$

$i\sqrt{2}$ $\quad 2i\sqrt{3}$ $\quad 2 \cdot 6$ $\quad 6i$

$2 \cdot 3$



Complex Numbers: Imaginary numbers and real numbers together. $a + bi$

You can use the complex number plane to represent a single complex number geometrically. Locate the real part of the number on the horizontal axis and the imaginary part of the number on the vertical axis. Graph 3-4i the same way you would graph (3, -4)

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Simplify. Write in the form $a + bi$

$\sqrt{-18} + 7$ $\qquad \sqrt{-121} - 7$

18

$\sqrt{-1 \cdot 18}$

$2 \cdot 9$ $\quad 7 + 3i\sqrt{2}$ $\qquad -7 + 11i$

$3 \cdot 3$

Absolute Value of a Complex Number-distance from the origin on the complex number plane. You can find the absolute value using Pythagorean Theorem. $|a + bi| = \sqrt{a^2 + b^2}$

a

c $\quad$ b

Find

4i

$|4i|$ $\qquad |6 - 4i|$ $\qquad |-2 + 5i|$

$\sqrt{0^2 + 4^2}$ $\qquad \sqrt{6^2 + 4^2}$ $\qquad \sqrt{2^2 + 5^2}$

$\sqrt{16}$ $\qquad \sqrt{36 + 16}$ $\qquad \sqrt{4 + 25}$

4 $\qquad \sqrt{52} = 2\sqrt{13}$ $\qquad \sqrt{29}$

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If the sum of two complex numbers is 0, then each number is the opposite, or *additive inverse*, of the other. The additive inverse of a is -a.

Find the additive inverse of:

$-5i$ $\qquad 4 - 3i$ $\qquad a + bi$

$5i$ $\qquad -4 + 3i$ $\qquad -a - bi$

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When simplifying expressions, treat i as if it were a variable.
Add/subtract complex numbers–combine the real parts and combine the imaginary parts.

Simplify:

$(8+3i)+(-2+{}^{-}4i)$

$6-i$

$7-(3+2i)$

$7+(-3-2i)$

$4-2i$

$(3+6i)-(4-8i)$

$(3+6i)+(-4+8i)$

$-1+14i$

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Multiplying complex numbers–multiply as you would real numbers or binomials. HINT: $i^2=-1$

Simplify:

$(12i)(7i)$

$84i^2$

$84(-1)$

-84

$(6-5i)(4-3i)$

	6	$-5i$
4	24	$-20i$
$-3i$	$-18i$	$15i^2 \to -15$

$9-38i$

$(5-4i)^2$

$25-40i+16i^2$

$25-40i-16$

$9-40i$

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Solve:

E
M ↑

$4x^2=-100$

$\frac{4x^2}{4}=\frac{-100}{4}$

$\pm\sqrt{x^2}=\pm\sqrt{-25}$

$x=\pm 5i$

E
M
A ↑

$3x^2+48=0$

$-48\quad -48$

$\frac{3x^2}{3}=\frac{-48}{3}$

$\pm\sqrt{x^2}=\pm\sqrt{-16}$

$x=\pm 4i$

$-5x^2-150=0$

$-5x^2=150$

$\pm\sqrt{x^2}=\pm\sqrt{-30}$

$\sqrt{-1\cdot 30}$

$x=\pm i\sqrt{30}$

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Functions of the form $f(z)=z^2+c$ where c is a complex number generate fractal graphs on the complex plane. To test if z belongs to the graph, use z as the first input value and repeatedly use each output as the next input. If the output values do not become infinitely large, then z is on the graph.

Find the first three output values for:

$f(z)=z^2+i$ cycles; z on graph

$f(0)=0^2+i=i$

$f(i)=i^2+i=-1+i$

$f(-1+i)=(-1+i)^2+i$

$1-2i+\cancel{i^2}^{-1}+i$

$-i$

$f(z)=z^2-1+i$ grows; z not on graph

$f(0)=0^2-1+i=-1+i$

$f(-1+i)=1-2i+i^2-1+i$

$-1-i$

$f(-1-i)=1+2i+i^2-1+i$

$-1+3i$

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homework:

page 274 # 4-48 x 4, 46, 58-66 even