6.8 The Binomial Theorem

You have learned to multiply binomials using the Distributive Property.

If you are raising a *single* binomial to a power, you have another option for finding the product.

Consider:

Exponents of $(a + b)$

Exponent 0 – anything raised to the 0 power = 1 $(a + b)^0 = 1$

Exponent 1 – the original value is unchanged $(a + b)^1 = a + b$

Exponent 2 – multiply by itself $(a + b)^2 = a^2 + 2ab + b^2$

Exponent 3 – multiply again $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Look at the patterns: Exponents of a: $a^3 + 3a^2b + 3ab^2 + b^3$ Exponents of b: $a^3 + 3a^2b + 3ab^2 + b^3$

Now lets look at the coefficients

$$1$$
$$a + b$$
$$a^2 + 2ab + b^2$$
$$a^3 + 3a^2b + 3ab^2 + b^3$$

Pascal's Triangle – A triangular array of numbers that give the coefficients for a single binomial being raised to a power.

Pascal's Triangle Formation:

1. Line the border with 1's
2. Place the sum of the two adjacent numbers within a row between and underneath the two original numbers.

Coefficients of an Expansion (Pascal's Triangle)

$$(a+b)^0 \longrightarrow 1$$
$$(a+b)^1 \longrightarrow 1 \quad 1$$
$$(a+b)^2 \longrightarrow 1 \quad 2 \quad 1$$
$$(a+b)^3 \longrightarrow 1 \quad 3 \quad 3 \quad 1$$
$$(a+b)^4 \longrightarrow 1 \quad 4 \quad 6 \quad 4 \quad 1$$
$$(a+b)^5 \longrightarrow 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$$
$$(a+b)^6 \longrightarrow 1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1$$
$$(a+b)^7 \longrightarrow 1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1$$

Each row of Pascal's Triangle contains coefficients for the expansion of $(a + b)^n$.

Use Pascal's Triangle to expand. $(a + b)^8$

1 7 21 35 35 21 7 1

$$1a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8ab^7 + 1b^8$$

Use Pascal's Triangle to expand. $(x - 2)^4$

$$1x^4 + 4x^3(-2) + 6x^2(-2)^2 + 4x(-2)^3 + 1(-2)^4$$
$$x^4 - 8x^3 + 24x^2 - 32x + 16$$

Use Pascal's Triangle to expand. $(2x - 3)^3$

$$1(2x)^3 + 3(2x)^2(-3) + 3(2x)(-3)^2 + 1(-3)^3$$
$$8x^3 - 36x^2 + 54x - 27$$

You can also use combinations to help find the terms of a binomial expansion.

Binomial Theorem: For every positive integer n,

$(a + b)^n = {}_nC_0a^n + {}_nC_1a^{n-1}b + {}_nC_2a^{n-2}b^2 + {}_nC_3a^{n-3}b^3 \ldots + {}_nC_{n-1}ab^{n-1} + {}_nC_nab^n$

Notice that the sequence of exponents decreases for a while it increases for b.

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Use the binomial theorem to expand: $(v + w)^9 =$

${}_9C_0v^9 \quad {}_9C_1v^8w \quad {}_9C_2v^7w^2 \quad {}_9C_3v^6w^3 \quad {}_9C_4v^5w^4 \quad {}_9C_5v^4w^5 \quad {}_9C_6v^3w^6 \quad {}_9C_7v^2w^7 \quad {}_9C_8vw^8 \quad {}_9C_9w^9$

$1v^9 + 9v^8w^1 + 36v^7w^2 + 84v^6w^3 + 126v^5w^4 + 126v^4w^5 + 84v^3w^6$
$+ 36v^2w^7 + 9vw^8 + w^9$

Use the binomial theorem to expand: $(c - 2)^5 =$

${}_5C_0c^5 \quad {}_5C_1c^4(-2) \quad {}_5C_2c^3(-2)^2 \quad {}_5C_3c^2(-2)^3 \quad {}_5C_4c(-2)^4 \quad {}_5C_5(-2)^5$

$1 \cdot c^5 + 5c^4(-2) + 10c^3(-2)^2 + 10c^2(-2)^3 + 5c(-2)^4 + 1(-2)^5$

$c^5 - 10c^4 + 40c^3 - 80c^2 + 80c - 32$

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Suppose an event has a probability of success p and a probability of failure q. Each term in expansion $(p + q)^n$ represents a probability. For example, ${}_{10}C_2p^8q^2$ represents the probability of eight successes in ten trials.

WNBA star Dawn Staley makes about 90% of the free throws she attempts.

$p = .9$
$q = .1$

- Find the probability that Dawn Staley will make exactly 9 out of 10 consecutive free throw attempts.

  ${}_{10}C_1\ p^9q^1$

  $10(.9)^9(.1)^1 = .387 \approx 39\%$

- Find the probability that Dawn Staley will make exactly 6 out of 10 consecutive free throw attempts.

  ${}_{10}C_4\,(p^6)(q^4)$ $(.9^6)(.1^4)$

  $210(.53)(.0001) = .0113 \approx 1\%$

- Find the probability that Dawn Staley will make exactly 10 out of 10 consecutive free throw attempts.

  ${}_{10}C_0\ p^{10}q^0$

  $1(.9)^{10} = .35 \approx 35\%$

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Find the probability that Dawn Staley will make exactly 7 out of 12 consecutive free throw attempts.

${}_{12}C_5\ p^7q^5$

$= 792(.9)^7(.1)^5$

$= .0038 \approx 0.4\%$

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To raise money, the club is selling raffle tickets. Each ticket has a 10% chance of winning a prize. You buy 7 tickets. To the nearest tenth of a percent, find the probability of winning exactly 1 time, find the probability of winning exactly 2 times.

$p = .1$

$q = .9$

${}_7C_6\, p^1 q^6$

$= 7(.1)^1(.9)^6$

$\approx .37 \approx 37\%$

${}_7C_5\, p^2 q^5$

$= 21\,(.1)^2(.9)^5$

$= .12 = 12\%$

homework:

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