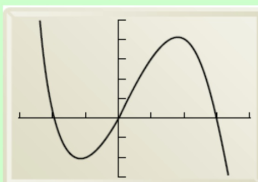
6.2 Polynomials and Linear Factors

A polynomial can be written as a product of linear factors.

Linear factor-the variable is linear and the factor is a binomial.

Linear factors help to find the zeros of a function (where it crosses the x-axis...roots, solutions, x-intercepts)



Write the following expressions as polynomials in standard form

$(x+1)(x+1)(x+2)$

	x	1
x	x^2	x
1	x	1

	x^2	$+2x$	$+1$
x	x^3	$2x^2$	x
2	$2x^2$	$4x$	2

x^3+4x^2+5x+2

$(x-1)(x+3)(x+4)$

x^2+2x-3

$x^3+6x^2+5x-12$

	x^2	$2x$	-3
x	x^3	$2x^2$	$-3x$
4	$4x^2$	$8x$	-12

You can sometimes use the GCF of the terms to help you factor the polynomial.

Write the polynomials in factored form.

$3x^3-3x^2-36x$

$3x(x^2-x-12)$

$3x(x-4)(x+3)$

$3x^3-18x^2+24x$

$3x(x^2-6x+8)$

$3x(x-4)(x-2)$

Several popular models of carry-on luggage have a length 10 inches greater than their depth. To comply with airline regulations, the sum of the length, width, and depth may not exceed 40 inches.

A. Assume that the sum of the length, width, and depth is 40 in. Graph the function relating volume V to depth x. Find the x-intercepts. What do they represent?

$\ell = (\cancel{d}x + 10)$

$40 = \ell + w + \cancel{d}$

$40 = (x+10) + w + x = 2x + 10 + w = 40$

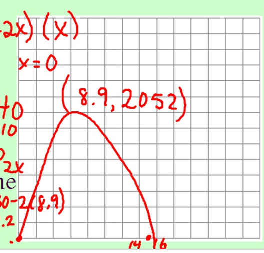
$-10 \qquad -10$

$2x + w = 30$

$w = 30 - 2x$

$V = (x+10)(30-2x)(x)$

$x=-10 \quad x=15 \quad x=0$



B. What is a reasonable domain?

$0 < x \le 15$

C. What is maximum volume? What are the corresponding dimensions?

$2052\ in^3$

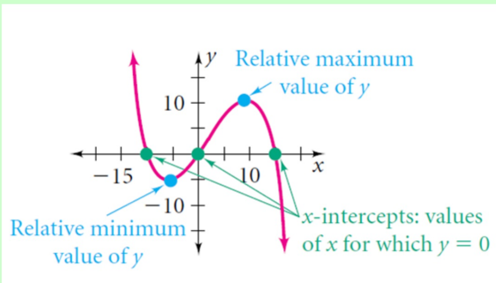
depth = 8.9

length = 18.9

$w = 30 - 2(8.9)$

$w = 12.2$

Relative Maximum (minimum)-The y-value of a point on the graph of a function that is higher (lower) than the nearby points.



Find the zeros.

$y=(x-7)(x-5)(x-3)$

$x=7 \quad x=5 \quad x=3$

$y=(x+1)(x-1)(x-3)$

$x=-1 \quad x=1 \quad x=3$

Factor Theorem

The expression x - a is a linear factor of a polynomial if and only if the value of a is a zero of the related polynomial function.

Write a polynomial in standard form from the following zeros

Zeros: -4, -2, -1

$y=(x+4)(x+2)(x+1)$

$y=x^2+6x+8$

x	x^3	$6x^2$	$8x$
1	x^2	$6x$	8

$y=x^3+7x^2+14x+8$

Zeros: 2, -3, 0

$y=(x-2)(x+3)x$

$y=(x^2+x-6)x$

$y=x^3+x^2-6x$

Multiple zero: a repeated zero. If a linear factor of a polynomial is repeated, then the zero is repeated.

Multiplicity: comes from a multiple zero; it is equal to the number of times the zero occurs.

For each function, find any multiple zeros and state the multiplicity

$f(x)=(x-2)(x+1)(x+1)^2$

$x=2 \quad x=-1 \quad x=-1$ multiplicity of 2

$f(x)=(x-2)(x+1)^3$

$x=2 \quad x=-1$ with a multiplicity of 3

$y=x^3-4x^2+4x$

$y=x(x^2-4x+4)$

$y=x(x-2)^2$

$x=0$

$x=2$ multiplicity 2

The factor theorem helps relate four key facts about a polynomial. These facts are all equivalent--that is, if you know one you know them all. $(x+4)(x-1)$

-4 is a solution of $x^2 + 3x - 4 = 0$

-4 is an x-intercept of the graph of $y = x^2 + 3x - 4$

-4 is a zero of $y = x^2 + 3x - 4$

x + 4 is a factor of $x^2 + 3x - 4$

homework:

pg 311 # 2-36 even, 15