6.3 Dividing Polynomials

Polynomial Division-will help find all the zeros of a polynomial function. There are two methods: Long division and synthetic division.

Recall that when a numerical division has a remainder of 0, the divisor and the quotient are factors of the dividend.

If numerical division leaves a remainder, neither the divisor nor the quotient is a factor of the dividend.

$$\begin{array}{r} 4 \leftarrow \text{Quotient} \\ \text{Divisor} \rightarrow 6\overline{)24} \\ \uparrow \\ \text{Dividend} \end{array}$$

$$\begin{array}{r} 7 \\ 8\overline{)56} \\ -56 \\ \hline 0 \end{array}$$

7 & 8 are factors of 56

$$\begin{array}{r} 8 \\ 5\overline{)42} \\ -40 \\ \hline 2 \end{array}$$

Neither 5 nor 8 is a factor of 42

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Division serves as a test of whether one polynomial is a factor of another.

Long Division-Divide into polynomials with the same process you use with whole numbers: Divide, Multiply, Subtract, Bring Down, repeat.

1. When there is no remainder, the polynomial is factored.
2. When there is a remainder, the polynomial is not a factor.

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Divide $x^2 - 3x + 1$ by $x - 4$

$$\begin{array}{r} x + 1 \quad r.\ 5 \\ x - 4\overline{)x^2 - 3x + 1} \\ -x^2 + 4x \quad\quad \\ \hline x + 1 \\ -x + 4 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 57 \\ 8\overline{)456} \\ -40 \\ \hline 56 \\ -56 \\ \hline 0 \end{array}$$

X - 4 is not a factor

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$(2x^2 - 19x + 24) \div (x - 8)$

$$\begin{array}{r} 2x - 3 \\ x-8\overline{)2x^2 - 19x + 24} \\ -2x^2 + 16x \quad\quad \\ \hline -3x + 24 \\ -3x + 24 \\ \hline 0 \end{array}$$

X-8 is a factor

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To divide by a linear factor, you can use a simplified process known as: Synthetic Division-omit all variables and exponents. Divide by the zero. (you will add instead of subtract).

Divide $3x^3 - 4x^2 + 2x - 1$ by $x + 1$

-1	3	-4	2	-1
	↓	-3	7	-9
	$3x^2$	$-7x$	9	r. -10

1. bring down
2. multiply
3. add

repeat 2 & 3...

4. put in variables & exponents

$x+1$ is not a factor

$x + 2\overline{)x^3 - 2x^2 - 5x + 6}$

-2	1	-2	-5	6
	↓	-2	8	-6
	$1x^2$	$-4x$	3	r. 0

$x+2$ is a factor

$(x+2)(x^2-4x+3) = x^3-2x^2-5x+6$

$(x+2)(x-1)(x-3)$ are all your factors

The volume in cubic feet of a shipping carton is $V(x) = x^3 - 6x^2 + 3x + 10$

The height is $x - 5$ feet.

A) Find the linear expressions for the other dimensions. (the length is greater than the width)

B) If the width of the carton is 4 feet, what are the other dimensions.

5	1	-6	3	10
	↓	5	-5	-10
	$1x^2$	$-1x$	-2	r. 0

$(x-5)(x^2-x-2) = V$

$\underset{h}{(x-5)}\underset{w}{(x-2)}\underset{\ell}{(x+1)} = V$

$w = x-2 = 4$

$+2 \quad +2$

$x = 6$

$h = 6-5 = 1$ ft

$\ell = 6+1 = 7$ ft

Divide $x^3 - 6x + 9$ by $x - 3$

3	1	0	-6	9
	↓	3	9	9
	$1x^2$	$3x$	3	r. 18

$x-3$ is not a factor

Find f(3) if $f(x) = x^3 - 6x + 9$

$3^3 - 6(3) + 9$

$27 - 18 + 9$

18

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Another method is to use the Remainder Theorem: If a polynomial P(x) of degree n 1 is divided by (x – a), where a is a constant, then the remainder is P(a). In other words, by using synthetic division, the remainder will be the answer.

Use synthetic division to find P(–1) for $P(x) = 2x^4 + 6x^3 - 5x^2 - 60$

$$\begin{array}{r|rrrrr} -1 & 2 & 6 & -5 & 0 & -60 \\ & \downarrow & -2 & -4 & 9 & -9 \\ \hline & 2 & 4 & -9 & 9 & -69 \end{array}$$

$P(-1) = -69$

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Use synthetic division to find P(3) for $P(x) = x^4 - 2x^3 + x - 9$

$$\begin{array}{r|rrrrr} 3 & 1 & -2 & 0 & 1 & -9 \\ & \downarrow & 3 & 3 & 9 & 30 \\ \hline & 1 & 1 & 3 & 10 & 21 \end{array}$$

the remainder is 21

$P(3) = 21$

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homework:

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