6.4 Solving Polynomial Equations

Solving Equations by Graphing

You can solve a polynomial equation by:

1. Graphing each side of the equation separately and finding the x value(s) at the point(s) of intersection.
2. Setting the equation equal to zero and finding the x-intercepts.

Solve: (use your gc)

$x^3 - 19x = -2x^2 + 20$

-5, -1, 4

$x^3 + 3x^2 = x + 3$

-3, -1, 1

The dimensions of a portable kennel are

width: x = 1.5 ft

length: x + 7 = 8.5 ft

height: x - 1 = 0.5 ft

$5.9 = x(x+7)(x-1)$

The volume is 5.9 ft^3. Find the dimensions in inches.

Solving equations by factoring:

Sum of cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Difference of cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

same, opposite, always positive

SOAP

Factor:

$8x^3 - 1$

$a = 2x$

$b = 1$

$(2x-1)(4x^2+2x+1)$

$216x^3 - 125$

$a = 6x$

$b = 5$

$(6x-5)(36x^2+30x+25)$

$27x^3 + 64$

$a = 3x$

$b = 4$

$(3x+4)(9x^2-12x+16)$

Solve:

$x^3 + 8 = 0$

$a = x$

$b = 2$

$(x+2)(x^2-2x+4) = 0$

$x = -2$

$1 \pm i\sqrt{3} = x$

$x = \frac{2 \pm \sqrt{2^2 - 4(1)(4)}}{2(1)}$

$\frac{2 \pm \sqrt{4-16}}{2}$

$\frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2i\sqrt{3}}{2} = 1 \pm i\sqrt{3}$

$27x^3 - 1 = 0$

$a = 3x$

$b = 1$

$(3x-1)(9x^2+3x+1)$

$x = \frac{1}{3}$

$x = \frac{1 \pm i\sqrt{3}}{6}$

$x = \frac{-3 \pm \sqrt{3^2 - 4(9)(1)}}{2(9)}$

$x = \frac{-3 \pm \sqrt{9-36}}{18}$

$x = \frac{-3 \pm \sqrt{-27}}{18} = \frac{3 \pm 3i\sqrt{3}}{18}$

$= \frac{1 \pm i\sqrt{3}}{6}$

You can sometimes factor a polynomial of a higher degree by using the techniques you have used in solving polynomials of a lower degree. Make a temporary substitution of variables. Rewrite and substitute $a = x^2$. Or, use the box with the correct exponents.

Factor:

$x^4 + 7x^2 + 6$

$a = x^2$

$a^2 + 7a + 6$

$(a+6)(a+1)$

$x^4 - 3x^2 - 10$

	x^2	2
x^2	x^4	$2x^2$
-5	$-5x^2$	-10

$(x^2-5)(x^2+2)$

$x^4 - 6x^2 - 27$

$(x^2-9)(x^2+3)$

$(x+3)(x-3)(x^2+3)$

Solve:

$x^4 + 11x^2 + 18 = 0$

	x^2	2
x^2	x^4	$2x^2$
9	$9x^2$	18

$(x^2+9)(x^2+2) = 0$

$x^2 + 9 = 0$

$x^2 = -9$

$\sqrt{x^2} = \pm\sqrt{-9}$

$x = \pm 3i$

$x^2 + 2 = 0$

$x^2 = -2$

$\sqrt{x^2} = \pm\sqrt{-2}$

$x = \pm i\sqrt{2}$

$x^4 - 4x^2 - 45 = 0$

	x^2	-9
x^2	x^4	$-9x^2$
5	$5x^2$	-45

$(x^2-9)(x^2+5) = 0$

$x^2 - 9 = 0$

$x^2 = 9$

$\sqrt{x^2} = \pm\sqrt{9}$

$x^2 = \pm 3$

$x^2 + 5 = 0$

$x^2 = -5$

$\sqrt{x^2} = \pm\sqrt{-5}$

$x = \pm i\sqrt{5}$

The width of a box is 2 m less than the length. The height is 1 m less than the length. The volume is 60 m^3. Find the dimensions of the box.

$5 = \ell$

$3 = \ell - 2$

$4 = \ell - 1$

$\ell(\ell-2)(\ell-1) = 60$

use GC, only positive length makes sense.

$x = 5$

Solve: $2x^3 + 2 = 0$

$2(x^3+1) = 0$

$a = x$

$b = 1$

$2(x+1)(x^2 - x + 1) = 0$

$x = -1$

$x = \frac{1 \pm i\sqrt{3}}{2}$

$\frac{1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$

$\frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{-3}}{2}$

$= \frac{1 \pm i\sqrt{3}}{2}$

homework

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