6.5 A Theorems about Roots and Polynomial Equations

ROOTS=ZEROS=SOLUTIONS=X-INTERCEPTS!

Both the constant term and the leading coefficient of a polynomial can play a key role in identifying the rational roots of the related polynomial equation.

Rational Root Theorem: If $\frac{p}{q}$ is in simplest form and is a rational root of the polynomial equation $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0 = 0$ with integer coefficients, then p must be a factor of a_0 and q must be a factor of a_n.

Equivalent equations:

standard form:

$24x^3 - 22x^2 - 5x + 6 = 0$

factored form:

$\left(x + \frac{1}{2}\right)\left(x - \frac{2}{3}\right)\left(x - \frac{3}{4}\right) = 0$

so the roots are $\frac{-1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$

The numerators (p) are all factors of the constant, 6 (a_0) and the denominators (q) are all factors of the leading coefficient, 24 (a_n)

You can use the Rational Root Theorem to find any rational roots of a polynomial equations with integer coefficients.

Find the rational roots of the following:

$x^3 - 4x^2 - 2x + 8 = 0$

$\dfrac{\text{factors of } 8}{\text{factors of } 1} : \dfrac{\pm 1 \quad \pm 2 \quad \pm 4 \quad \pm 8}{\pm 1}$

$\cancel{1}$: $1^3 - 4(1)^2 - 2(1) + 8$

$1 - 4 - 2 + 8 \neq 0$

$\cancel{-1}$ $(-1)^3 - 4(-1)^2 - 2(-1) + 8$

$-1 - 4 + 2 + 8 \neq 0$

$\cancel{2}$ $(2)^3 - 4(2)^2 - 2(2) + 8$

$8 - 16 - 4 + 8 \neq 0$

$\cancel{-2}$ $(-2)^3 - 4(-2)^2 - 2(-2) + 8$

$-8 - 16 + 4 + 8 \neq 0$

4 $4^3 - 4(4)^2 - 2(4) + 8$

$64 - 64 - 8 + 8 = 0$

$\cancel{-4}$ $(-4)^3 - 4(4)^2 - 2(-4) + 8$

$-64 - 64 + 8 + 8 \neq 0$

$\cancel{8}$ $8^3 - 4(8)^2 - 2(8) + 8$

$512 - 256 - 16 + 8 \neq 0$

$\cancel{-8}$ $-512 - 256 + 16 + 8 \neq 0$

Find the rational roots of the following:

$3x^3 - x^2 - 15x + 5 = 0$

$\dfrac{\text{factors of } 5}{\text{factors of } 3} : \dfrac{\pm 1 \quad \pm 5}{\pm 1 \quad \pm 3}$

use GC.

$\cancel{1}$ $\cancel{-8}$ $\cancel{-1}$ $\cancel{16}$ $\cancel{5}$ $\cancel{280}$ $\cancel{-5}$ $\cancel{-320}$ $\frac{1}{3}$ 0 $\cancel{-\frac{1}{3}}$ $\cancel{9\frac{7}{9}}$ $\cancel{\frac{5}{3}}$ $\cancel{-8\frac{8}{9}}$ $\cancel{-\frac{5}{3}}$ $\cancel{13\frac{1}{3}}$

root $x = \frac{1}{3}$

Use the Rational Root Theorem. Find the roots of the equation.

$x^3 - 2x^2 - 5x + 10 = 0$

factors of 10: $\pm 1 \quad \pm 2 \quad \pm 5 \quad \pm 10$

factors of 1: ± 1

$\cancel{\frac{1}{4}} \quad \cancel{\frac{-1}{12}} \quad \frac{2}{0} \quad \cancel{\frac{-2}{4}} \quad \cancel{\frac{5}{60}} \quad \cancel{\frac{-5}{-140}} \quad \cancel{\frac{10}{760}} \quad \cancel{\frac{-10}{-1140}}$

2 is a zero.

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -5 & 10 \\ & \downarrow & 2 & 0 & -10 \\ \hline & 1 & 0 & -5 & 0 \end{array}$$

$x^2 - 5$ is a factor. Solve for x to get 2 roots.

$(x^2 - 5) = 0$

$x^2 = 5 \qquad x = \pm\sqrt{5}$

Page 5

Use the Rational Root Theorem. Find all the roots.

$3x^3 + x^2 - x + 1 = 0$

factors of 1: ± 1

factors of 3: $\pm 1 \quad \pm 3$

$\cancel{\frac{1}{4}} \quad \frac{-1}{0} \quad \cancel{\frac{1/3}{8/9}} \quad \cancel{\frac{-1/3}{4/3}}$

$$\begin{array}{r|rrrr} -1 & 3 & 1 & -1 & 1 \\ & \downarrow & -3 & 2 & -1 \\ \hline & 3 & -2 & 1 & 0 \end{array}$$

$(x+1)(3x^2 - 2x + 1) = 0$

$x = \frac{2 \pm \sqrt{2^2 - 4(3)(1)}}{2(3)} \qquad x = \frac{2 \pm \sqrt{4 - 12}}{6}$

$x = \frac{2 \pm 2i\sqrt{2}}{6}$ simplify

$x = -1, \; x = \frac{1 \pm i\sqrt{2}}{3}$

Page 6

Use the Rational Root Theorem. Find all roots.

$2x^3 - x^2 + 2x - 1 = 0$

factors of 1: ± 1

factors of 2: $\pm 1 \quad \pm 2$

1	2
-1	-6
$\frac{1}{2}$	0
$-\frac{1}{2}$	-2.5

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & 2 & -1 \\ & \downarrow & 1 & 0 & 1 \\ \hline & 2 & 0 & 2 & 0 \end{array}$$

$2x^2 + 2$

$(2x-1)(2x^2+2) = 0$

$x = \frac{1}{2} \qquad 2x^2 = -2$

$x^2 = -1$

$x = \pm i$

Page 7

Use the Rational Root Theorem. Find all roots.

$10x^4 + x^3 + 7x^2 + x - 3 = 0$

factors of 3: $\pm 1 \quad \pm 3$

factors of 10: $\pm 1 \quad \pm 2 \quad \pm 5 \quad \pm 10$

$x = \frac{1}{2} \qquad (2x - 1)$

$x = -\frac{3}{5} \qquad (5x + 3)$

product is $10x^2 + x - 3$

$(x^2 + 1) = 0$

$x^2 = -1$

$x = \pm i$

divide poly by factor to get another factor

$$\begin{array}{r} x^2 + 1 \\ 10x^2 + x - 3 \overline{) \, 10x^4 + x^3 + 7x^2 + x - 3} \\ \underline{10x^4 + x^3 - 3x^2} \\ 10x^2 + x - 3 \end{array}$$

Solve for x.

Page 8

homework:

page 333 # 1-12