6.5 B Theorems about Roots & Polynomial Equations

We found irrational solutions to quadratic equations.

For example, by the quadratic formula, the solutions of $x^2-4x-1=0$ are $2+\sqrt{5}$ and $2-\sqrt{5}$

Conjugates: Number pairs in the form $a+\sqrt{b}$ and $a-\sqrt{b}$

Irrational roots ALWAYS come in pairs!

Irrational Root Theorem: Let a and b be rational numbers, and let $\sqrt{b}$ be an irrational number. If $a+\sqrt{b}$ is a root of a polynomial equation with rational coefficients, then the conjugate $a-\sqrt{b}$ is also a root.

A polynomial equation with rational coefficients has the roots $2-\sqrt{7}$ and $\sqrt{5}$. Find two additional roots.

$2+\sqrt{7}$ $\quad -\sqrt{5}$

A polynomial equation with rational coefficients has the roots $3+\sqrt{10}$ and $-\sqrt{3}$ Find two additional roots.

$3-\sqrt{10}$ $\quad \sqrt{3}$

One of the roots of a polynomial equation is $4-\sqrt{2}$

Can you be certain that $4+\sqrt{2}$ is also a root of the equation?

No. You must know the polynomial has rational coefficients

Complex Conjugates: the number pairs of the form $a+bi$ and $a-bi$

The product of complex conjugates is a^2+b^2.

	a	bi
a	a^2	$\cancel{abi}$
$-bi$	$\cancel{-abi}$	$-b^2i^2$

a^2+b^2

$-b^2(-1)=b^2$

You can use complex conjugates to find an equation's imaginary roots.
Imaginary Roots ALWAYS come in pairs!
Imaginary Root Theorem: If the imaginary number $a+bi$ is a root of a polynomial equation with real coefficients, then the conjugate $a-bi$ is also a root.

If a polynomial equation with real coefficients has $3i$ and $-2+i$ among its roots, then what two other roots must it have?

$-3i \qquad -2-i$

$\sqrt{-9} = \pm 3i$

Describe the degree of this equation.

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Find a third degree polynomial equation with rational coefficients that has roots -1 and $2-i$, $2+i$

$(x+1)\ (x-(2-i))\ (x-(2+i))$

	x	-2	$+i$
x	x^2	$-2x$	$\cancel{ix}$
-2	$-2x$	4	$\cancel{-2i}$
$-i$	$\cancel{-ix}$	$\cancel{+2i}$	$-i^2=1$

x^2-4x+5

	x^2	$-4x$	$+5$
x	x^3	$-4x^2$	$5x$
1	x^2	$-4x$	5

x^3-3x^2+x+5

Find a fourth degree polynomial equation with rational coefficients that has roots i and $2i$, $-i$, $-2i$

$(x-i)(x+i)(x-2i)(x+2i)$

$x^2+1 \qquad x^2+4$

x^4+5x^2+4

Find a fourth degree polynomial equation with integer coefficients that has roots $\sqrt{3}$ and $1-i$, $-\sqrt{3}$, $1+i$

$(x-\sqrt{3})(x+\sqrt{3}) \qquad (x-(1-i))\ (x-(1+i))$

DOS → x^2-3

	x	-1	$+i$
x	x^2	$-x$	$\cancel{ix}$
-1	$-x$	1	$\cancel{-i}$
$-i$	$\cancel{-ix}$	$\cancel{i}$	$-i^2=1$

	x^2	$-2x$	$+2$
x^2	x^4	$-2x^3$	$+2x^2$
-3	$-3x^2$	$6x$	-6

$x^4-2x^3-x^2+6x-6$

homework:
page 333 # 13-29 odd and 20