6.6 The Fundamental Theorem of Algebra

Find the solutions of $x^4 - 5x^2 + 4 = 0$

$$(x^2-4)(x^2-1)=0$$

$\pm 2 \quad \pm 1$

Identify each solution as real or imaginary

How many solutions are there? 4

Find the solutions of $x^4 + 7x^2 + 12 = 0$

$$(x^2+3)(x^2+4)=0$$

$\pm i\sqrt{3} \quad \pm 2i$

Identify each solution as real or imaginary

How many solutions are there? 4

Make a conjecture about the number of zeros of a fourth-degree polynomial function regardless of the type of zeros. 4

You have solved polynomial equations and found that their roots are included in the set of complex numbers. That is, the roots have been integers, rational numbers, irrational numbers, and imaginary numbers.

Fundamental Theorem of Algebra: The roots of every polynomial equation, even those with imaginary coefficients, are complex numbers. Including imaginary roots, and multiple roots, an nth degree polynomial equation has exactly n roots; the related polynomial function has exactly n zeros.

You can factor a polynomial of degree n into n linear factors. The number n includes multiple roots.

State the number of complex roots (all answers), the possible number of real roots and the possible rational roots.

$$x^3 + 2x^2 - 4x - 6 = 0$$

possible complex: 3

possible real: 3 (0 imag)

1 (2 imaginary)

possible rational roots: $\dfrac{\text{factors of } 6}{\text{factors of } 1}$: $\pm 1, \pm 2, \pm 3, \pm 6$

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State the number of complex roots (all answers), the possible number of real roots and the possible rational roots.

$$x^4 - 3x^3 + x^2 - x + 3 = 0$$

4

$\pm 1, \; \pm 3$

real 4, 2, 0

imaginary 0, 2 4

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Find all the zeros of the function. $f(x) = x^3 + x^2 - x + 2$

$$\frac{\text{factors of } 2}{\text{factors of } 1} \qquad \frac{\pm 1 \;\; \pm 2}{\pm 1}$$

-2

$\cancel{-1}$

$\cancel{1}$

$\cancel{2}$

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -1 & 2 \\ & \downarrow & -2 & 2 & -2 \\ \hline & 1 & -1 & 1 & 0 \end{array}$$

$$x^2 - x + 1 = 0$$

$$\frac{1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)} = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm i\sqrt{3}}{2}$$

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Find all the zeros of the function. $y = x^5 + 3x^4 - x - 3$

-3 $(x+3)$

-1 $(x+1)$

1 $(x-1)$

$\cancel{3}$

$$> (x^2-1)(x+3)(x^2+1) = 0$$

	x^2	-1
x	x^3	$-x$
3	$3x^2$	-3

$$\begin{array}{r} x^2 + 1 \\ x^3 + 3x^2 - x - 3 \overline{) \, x^5 + 3x^4 + 0x^3 + 0x^2 - x - 3} \\ -x^5 + 3x^4 + x^3 + 3x^2 \\ \hline x^3 + 3x^2 - x - 3 \\ x^3 + 3x^2 - x - 3 \\ \hline 0 \end{array}$$

$x^2 + 1 = 0$

$x^2 = -1$

$\sqrt{x^2} = \pm\sqrt{-1}$

$x = \pm i$

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homework:

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