6.7 Permutations and Combinations

The word "combination" is used loosely without thinking if the order of things matter. For example:

My fruit salad is a combination of apples, grapes and bananas.

The combination to the safe is 583.

If the order does NOT matter, it is a Combination

If the order DOES matter, it is a Permutation

Permutation – An arrangement of items in a particular order.

– We reduce the number of available choices each time.

– Permutation = Position

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Find the number of Permutations by using:

1. **Multiplication Counting Principle**: Multiply the number of ways to pick each item.
2. **n Factorial**: $n! = n(n-1)(n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$. For $n = 1$, $n! = 1$

In how many different orders can ten dogs line up to be groomed?

$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 10! = 3{,}628{,}800$

How many ways can you arrange six trophies on a shelf?

$6! = 720$

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Some permutations do not use all the items available.

Number of Permutations: The number of permutations of n items of a set arranged r items at a time is ${}_nP_r$. ${}_nP_r = \frac{n!}{(n-r)!}$ for $0 \le r \le n$

${}_{10}P_4 = \frac{10!}{(10-4)!}$ ${}_{24}P_6 = 96{,}909{,}120$

$= \frac{10!}{6!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!}} = 5040$

How many arrangements of first, second, and third places are possible with ten yachts? ${}_{10}P_3 = 720$

How many four letter codes can be made if no letter can be used twice? ${}_{26}P_4 = 358{,}800$

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Combination – A selection of items in which order does not matter.

Number of Combinations: The number of combinations of n items of a set chosen r items at a time is ${}_nC_r$. ${}_nC_r = \frac{n!}{r!(n-r)!}$ for $0 \le r \le n$

${}_{10}C_4 = \frac{10!}{4!\,(10-4!)}$ ${}_{25}C_7 = 480{,}700$ ${}_8C_2 = 28$

$= \frac{10!}{4!\,(6!)} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6!}}{4!\,\cancel{6!}} = \frac{5040}{24} = 210$

A disk jockey wants to select 5 songs from a new CD that contains 12 songs. How many possible selections are possible?

${}_{12}C_5 = 792$

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A pizza menu allows you to select 4 toppings at no extra charge from a list of 9 possible toppings. In how many ways can you select 4 or fewer toppings?

$_9C_4 = 126$
$_9C_3 = 84$
$_9C_2 = 36$
$_9C_1 = 9$
$_9C_0 = 1$

256

Ten candidates are running for three seats in the student government. You may vote for as many as three candidates. In how many ways can you vote for three or fewer candidates? In how many ways can you vote for five or fewer people?

176 ; 638

homework:

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