

## 7.5 Solving Radical Equations

Radical Equations are equations that have variables in a radicand or have variables with rational exponents.

Examples:

$$3 + \sqrt{x} = 10 \quad (x - 2)^{\frac{2}{3}} = 25$$

Non-example:

$$\sqrt{3 + x} = 10$$

To solve a radical equation, isolate the radical on one side of the equation and then raise both sides to the same power.

If  $\sqrt[n]{x} = k$ , then  $(\sqrt[n]{x})^n = k^n$  and  $x = k^n$

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Solve.

$$\sqrt{5x+1} - 6 = 0$$
$$(\sqrt{5x+1})^2 = (6)^2$$

$$5x + 1 = 36$$

$$-1 \quad -1$$

$$5x = 35$$

$$x = 7$$

$$-10 + \sqrt{2x+1} = -5$$
$$+10 \quad +10$$

$$(\sqrt{2x+1})^2 = (5)^2$$

$$2x + 1 = 25$$

$$2x = 24$$

$$x = 12$$

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You can solve equations of the form  $x^{\frac{m}{n}} = k$  by raising each side of the equation to the reciprocal power,  $\frac{n}{m}$

In n is even, then  $((x)^{\frac{m}{n}})^{\frac{n}{m}} = |x|$ . If n is odd, then  $((x)^{\frac{m}{n}})^{\frac{n}{m}} = x$

If  $x^n = k$ , then  $(x^n)^{\frac{1}{n}} = k^{\frac{1}{n}}$  and  $x = k^{\frac{1}{n}}$

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Solve.

$$\frac{2(x+3)^{\frac{3}{2}}}{2} = \frac{54}{2}$$
$$\left[ (x+3)^{\frac{3}{2}} \right]^{\frac{2}{3}} = (27)^{\frac{2}{3}}$$

$$x + 3 = \sqrt[3]{27^2}$$

$$x + 3 = 3^2$$

$$x + 3 = 9$$

$$x = 6$$

$$\frac{3(x+1)^{\frac{3}{5}}}{3} = \frac{24}{3}$$
$$\left[ (x+1)^{\frac{3}{5}} \right]^{\frac{5}{3}} = (8)^{\frac{5}{3}}$$

$$x + 1 = \sqrt[3]{8^5}$$

$$x + 1 = 2^5$$

$$x + 1 = 32$$

$$x = 31$$

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An artist wants to make a plastic sphere for a sculpture. The plastic weighs **0.8 ounces per cubic inch**. The maximum weight of the sphere is to be **80 pounds**. What is the maximum radius the sphere can have?

$$16 \text{ oz} = 1 \text{ pound}$$

$$1280 \text{ oz} = 80 \text{ pounds}$$

$$1280 \text{ oz} \cdot \frac{\text{in}^3}{0.8 \text{ oz}} = 1600 \text{ in}^3$$

$$V = \frac{4}{3} \pi r^3$$

$$1600 = \frac{4}{3} \pi r^3$$

$$1200 = \pi r^3$$

$$\sqrt[3]{381.97} = \sqrt[3]{r^3}$$

$$7.26 \text{ in} = r$$

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Solve and check (for extraneous solutions).

$$\sqrt{5x-1} + 3 = x$$

$$(\sqrt{5x-1})^2 = (x-3)^2$$

$$5x-1 = x^2 - 6x + 9$$

$$0 = x^2 - 11x + 10$$

$$0 = (x-1)(x-10)$$

$$\text{extraneous} \rightarrow x=1, \quad x=10$$

$$\sqrt{x+2} - 3 = 2x$$

$$(\sqrt{x+2})^2 = (2x+3)^2$$

$$x+2 = 4x^2 + 12x + 9$$

$$0 = 4x^2 + 11x + 7$$

4x	4x
7x	7

$$(x+1)(4x+7)$$

$$x = -1, \quad x = -\frac{7}{4} = -1.75$$

extraneous

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Solve and check.

$$\sqrt{3x+2} - \sqrt{2x+7} = 0$$

$$+\sqrt{2x+7} = \sqrt{2x+7}$$

$$(\sqrt{3x+2})^2 = (\sqrt{2x+7})^2$$

$$3x+2 = 2x+7$$

$$x = 5$$

$$(x+1)^{\frac{2}{3}} - (9x+1)^{\frac{1}{3}} = 0$$

$$+(9x+1)^{\frac{1}{3}} = (9x+1)^{\frac{1}{3}}$$

$$\left((x+1)^{\frac{2}{3}}\right)^3 = \left((9x+1)^{\frac{1}{3}}\right)^3$$

$$(x+1)^2 = 9x+1$$

$$x^2 + 2x + 1 = 9x + 1$$

$$x^2 - 7x = 0$$

$$x(x-7) = 0$$

$$x=0, \quad x=7$$

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Solve and check.

$$\left((4x+3)^{\frac{2}{3}}\right)^3 = \left((16x+44)^{\frac{1}{3}}\right)^3$$

$$(4x+3)^2 = 16x+44$$

$$16x^2 + 24x + 9 = 16x + 44$$

$$16x^2 + 8x - 35 = 0$$

$$\frac{-8 \pm \sqrt{8^2 - 4(16)(-35)}}{2(16)} = \frac{-8 \pm \sqrt{2304}}{32} = \frac{-8 + 48}{32}$$

$$= 1.25$$

$$= \frac{-8 - 48}{32}$$

$$= -1.75$$

extr.

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homework.

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