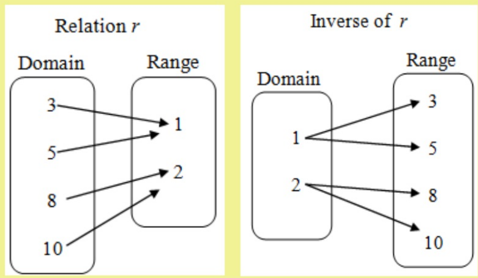
7.7 Inverse Relations and Functions

If a relation pairs element a of its domain to element b of its range, the inverse relation pairs b with a. So, if (a, b) is an ordered pair of a relation, then (b, a) is an ordered pair of its inverse.

The diagram shows a relation r and its inverse. The range of the relation is the domain of the inverse. The domain of the relation is the range of the inverse.



Find the inverse of relation s.

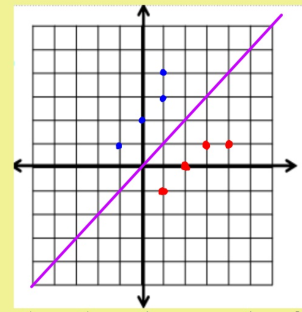
Relation s

x	1	2	3	4
y	-1	0	1	1

Inverse s

x	-1	0	1	1
y	1	2	3	4

Graph both
(use different colors)



vertical line test

Is the relation a function? yes

Is the inverse a function? no

Explain.

Describe how the line y = x is related to the graph of s and its inverse.

As shown on the last page, the graph of the inverse of a relation is the reflection over the line y = x of the graph of the relation. If a relation or function is described by an equation in x and y, you can interchange x and y to get the inverse and solve for y.

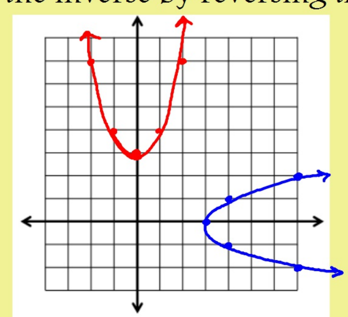
Find the inverse of $y = x^2 + 3$

$$x = y^2 + 3$$
$$-3 \qquad -3$$
$$x - 3 = y^2$$
$$\sqrt{x-3} = \sqrt{y^2}$$
$$\pm\sqrt{x-3} = y$$

Graphing a relation and its inverse.

Graph $y = x^2 + 3$ and its inverse. The graph of $y = x^2 + 3$ is a parabola that opens upward with a vertex (0, 3). The reflection of the parabola over the line y = x is the graph of the inverse. You can also find points on the graph of the inverse by reversing the coordinates of the points on $y = x^2 + 3$

$y = (x+0)^2 + 3$



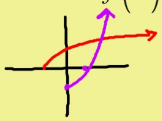
Does $y = x^2 + 3$ define a function? yes

Is its inverse a function? no

Explain. the inverse fails the vertical line test

The inverse of a function is denoted by f^{-1}. Read as "the inverse of f" or as "f inverse". The notation f(x) is used for functions, but the relation f^{-1} may not even be a function.

Consider the function $f(x) = \sqrt{x+1}$ Find the domain and range of f(x).



d: $-1 \le x < \infty$ $[-1, \infty)$ $x \ge -1$

r: $0 \le y < \infty$ $[0, \infty)$ $y \ge 0$

Find f^{-1}

$x = \sqrt{y+1}$

$x^2 = y+1$

$x^2 - 1 = y$

Find the domain and range of f^{-1}

d: $0 \le x < \infty$ $[0, \infty)$ $x \ge 0$

r: $-1 \le y < \infty$ $[-1, \infty)$ $y \ge -1$

Is a function? Explain.

Functions that model real-life situations are frequently expressed as formulas with letters that remind you of the variables they represent. When finding the inverse of a formula, it would be very confusing to interchange the letters. Keep the letters the same and just solve the formula for the other variable.

The function $d = \frac{r^2}{24}$ is a model for the distance d in feet that a car with locked brakes skids in coming to a complete stop from a speed of r mph. Find the inverse of the function. What is the best estimate of the spped of a car that makes skid marks 114 feet long?

$d = \frac{r^2}{24}$

$24d = r^2$

$\sqrt{24d} = r$

$\sqrt{24(114)} = 52$ mph

Find the inverse of the relation. Is the relation a function? yes

Is the inverse a function? no

x	−1	0	1	2
y	−2	−1	−1	−2

inverse

x	-2	-1	-1	-2
y	-1	0	1	2

Find the inverse of $y = -x^2 - 2$

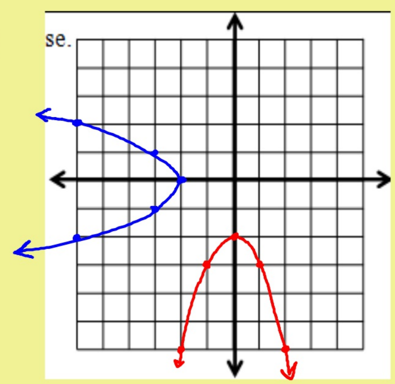
$x = -y^2 - 2$

$x + 2 = -y^2$

$-x - 2 = y^2$

$\pm\sqrt{-x-2} = y$

Graph $y = -x^2 - 2$ and its inverse.



The function $d = 16t^2$ models the distance d in feet that an object falls in t seconds. Find the inverse of the function. Use the inverse to estimate the time it takes an object to fall 50 ft.

$d = 16t^2$

$\frac{d}{16} = t^2$

$\sqrt{\frac{d}{16}} = t$

$\sqrt{\frac{50}{16}} = t$

$1.77 = t$

Consider the function $f(x) = \sqrt{2x + 1}$

Find the domain and range of f.

d: $-\frac{1}{2} \le x < \infty$ $[-\frac{1}{2}, \infty)$ $x \ge -\frac{1}{2}$

r: $0 \le y < \infty$ $[0, \infty)$ $y \ge 0$

Find f^{-1}

$x = \sqrt{2y+1}$

$x^2 = 2x + 1$

$x^2 - 1 = 2x$

$\frac{1}{2}x^2 - \frac{1}{2} = x$

Find the domain and range of f^{-1}

d: $0 \le x < \infty$ $[0, \infty)$, $x \ge 0$

r: $-\frac{1}{2} \le y < \infty$ $[-\frac{1}{2}, \infty)$ $y \ge -\frac{1}{2}$

Is f^{-1} a function? Explain.

yes

homework:

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