8.1 Exploring Exponential Models

Exponential Function: uses a variable for the exponent. $y = a(b)^x$

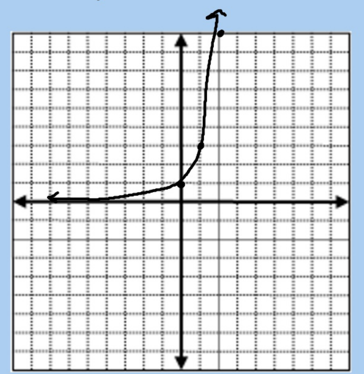
where x is a real number, $a \neq 0$, $b > 0$ and $b \neq 1$. a is the initial value (the y-intercept) and b is the common ratio.

Asymptote: a line the graph approaches, but never reaches, as it moves away from the origin.

Graph the following by making a table. Identify the asymptote.

$y = 3^x$

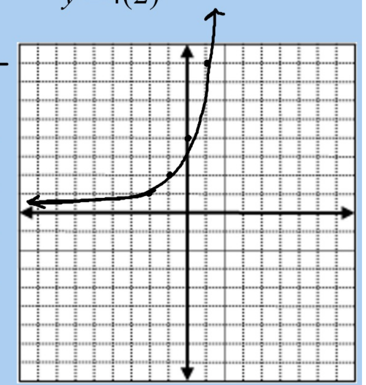
x	y
-2	1/9
-1	$\frac{1}{3}$
0	1
1	3
2	9



asymptote $y=0$

$y = 4(2)^x$

x	y
-2	1
-1	2
0	4
1	8
2	16

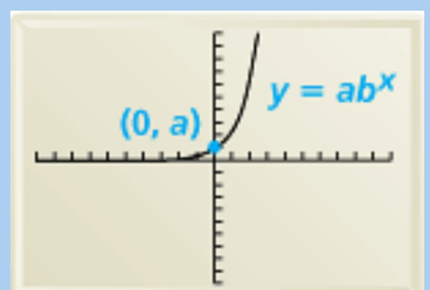


asymptote $y=0$

Exponential Growth

$b > 1$, graph curves up.

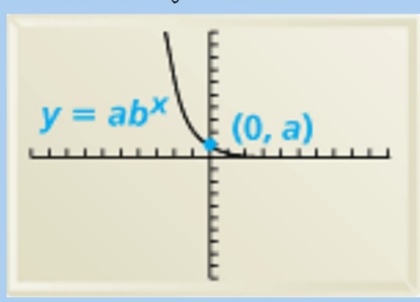
b is a growth factor.



Exponential Decay

$b < 1$, graph curves down

b is a decay factor



<u>Percent Increase</u>: $b = 1 + r$, where r is the rate of increase. You can use an exponential function to model population growth.

The population in the U.S. in 1994 was about 260 million with an average annual rate of increase of about 0.7%.

A) Find the growth factor. 0.7% $\xrightarrow{\text{decimal}}$ $.007 + 1 = 1.007$

B) Write an equation to model U.S. population growth. $y = 260{,}000{,}000(1.007)^x$

C) Predict the U.S. population in 2020 to the nearest million. $y = 260{,}000{,}000(1.007)^{26} = 312$ million

D) Why might this not be valid?

E) Suppose the rate changed to 1.4%. Write a function for this rate and predict the population in 2020.

$y = 260{,}000{,}000(1.014)^{26} = 373$ million

Write an exponential function $y = ab^x$ for a graph that include the following points.

(2, 4) and (3, 16)

$$\frac{16 = ab^3}{4 = ab^2}$$

$$\frac{16}{4} = \frac{a}{a} \quad \frac{b^3}{b^2}$$

$$4 = 1 \cdot b$$

$$4 = b$$

$$16 = a \cdot 4^3$$

$$\frac{16}{64} = \frac{64a}{64}$$

$$\frac{1}{4} = a$$

$$y = \frac{1}{4}(4)^x$$

(1, 6) and (0, 2)

$$\frac{6 = ab^1}{2 = ab^0}$$

$$3 = b$$

$$2 = a$$

$$y = 2(3)^x$$

Write an exponential function $y = ab^x$ for a graph that include the following points.

(2, 2) and (3, 4)

$$\frac{4 = ab^3}{2 = ab^2}$$

$$2 = 1 \cdot b$$

$$2 = b$$

$$2 = a \cdot 2^2$$

$$2 = 4a$$

$$1/2 = a$$

$$y = \frac{1}{2}(2)^x$$

<u>Decay Factor</u> – In an exponential function when $0 < b < 1$, b is the decay factor.

<u>Depreciation</u> – The decline in an item's value resulting from age and wear. When an item loses about the same percent of its value each year, you can use an exponential function to model the depreciation.

<u>Percent Decrease</u>: b = 1 – r where r is the rate of decrease

Determine whether the functions are exponential growth or decay.

$y = 100(0.12)^x$

decay

$.12 < 1$

$y = 0.2(5)^x$

growth

$5 > 1$

$y = 16\left(\frac{1}{2}\right)^x$

decay

$\frac{1}{2} < 1$

Suppose you want to buy a car that costs $11,800. The expected depreciation of the car is 20% per year.

A) What is the decay factor? $100\% - 20\% = 80\% = .8$

B) Write an equation to model the depreciation of the car.

$y = 11,800(.8)^x$

C) Estimate the depreciated value of the car after 6 years.

$y = 11,800(.8)^6 = \$3,093.30$

$(0, 20,000)$

The initial value of a car is $20,000. After one year the value of the car is about $17,000. Write an equation and then estimate the value after 6 years. $(1, 17000)$

$$\frac{17000 = ab^1}{20000 = ab^0}$$

$.85 = 1 \cdot b^1$

$.85 = b$

$a = y\text{-intercept} = 20000$

$y = 20000(.85)^x$

$y = 20000(.85)^6$

$y = \$7542.99$

homework:

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