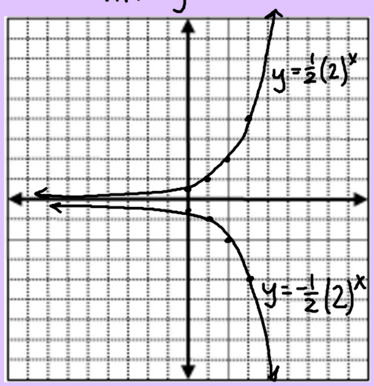
8.2 Properties of Exponential Functions

Graph. $y = \frac{1}{2}(2)^x$ $\quad y = -\frac{1}{2}(2)^x$ State the asymptotes

HA $y = 0$

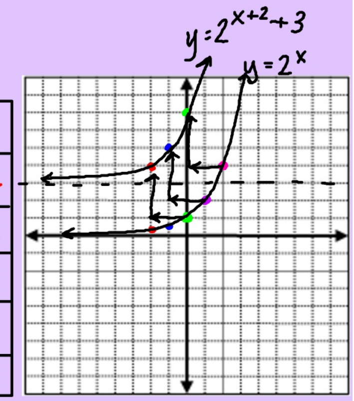
x	$y = \frac{1}{2}(2)^x$	$y = -\frac{1}{2}(2)^x$
-2	$\frac{1}{2}(2)^{-2} = \frac{1}{2}(\frac{1}{4}) = \frac{1}{8}$	$-\frac{1}{8}$
-1	$\frac{1}{2}(2)^{-1} = \frac{1}{2}(\frac{1}{2}) = \frac{1}{4}$	$-\frac{1}{4}$
0	$\frac{1}{2}$	$-\frac{1}{2}$
1	$\frac{1}{2}(2)^{1} = \frac{1}{2}(2) = 1$	-1
2	$\frac{1}{2}(2)^{2} = \frac{1}{2}(4) = 2$	-2
3	4	-4



When a is negative, what does the graph do? reflects over x-axis

Graph $y = 2^x$ and $y = 2^{x+2} + 3$

x	$y = 2^x$	$y = 2^{x+2} + 3$
-2	$\frac{1}{4}$	$2^{-2+2} + 3 = 1 + 3 = 4$
-1	$\frac{1}{2}$	$2^{-1+2} + 3 = 2 + 3 = 5$
0	1	$2^{0+2} + 3 = 4 + 3 = 7$
1	2	$2^{1+2} + 3 = 8 + 3 = 11$
2	4	$2^{2+2} + 3 = 16 + 3 = 19$



The graph of $y = ab^{x-h} + k$ is the graph of $y = ab^x$ translated h units horizontally and k units vertically.

Besides using exponentials to model population growth and decay, we can use it to calculate interest.

Compound Interest: interest on money calculated on a regular basis.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

- Yearly
- Monthly
- Quarterly
- Daily
- Continuously

P = principal

r = rate

t = time in years

n = number of times compounded per year

A = amount in account

Sally invested $5000 in a savings account that had a 1.5% interest. If she left the money in there for 4 years, how much money will be in her account:

Compounded monthly?

$A = 5000\left(1 + \frac{.015}{12}\right)^{12 \cdot 4}$

$A = 5000(1.00125)^{48}$

$A = \$5308.98$

Compounded semi-annually?

$A = 5000\left(1 + \frac{.015}{2}\right)^{2 \cdot 4}$

$A = 5000(1.0075)^{8}$

$A = \$5307.99$

Half-Life: amount of time it takes for half of a radioactive substance to decay.

a = amount of material

h = half-life time

t = time

$$y = a\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

Arsenic-74 is used to locate brain tumors. It has a half-life of 17.5 days. Write an exponential decay function for a 90-mg sample. Find the amount remaining after 6 days.

$$y = 90\left(\frac{1}{2}\right)^{\frac{x}{17.5}}$$

$$y = 90\left(\frac{1}{2}\right)^{\frac{6}{17.5}}$$

$$y = 70.96 \text{ mg}$$

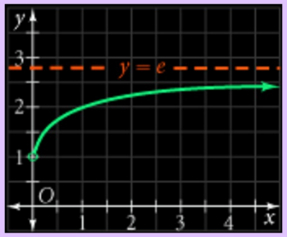
Technetium-99 has a half-life of 6 hours. Find the amount that remains from a 50-mg supply after 25 hours.

$$y = 50\left(\frac{1}{2}\right)^{\frac{x}{6}}$$

$$y = 50\left(\frac{1}{2}\right)^{\frac{25}{6}}$$

$$y = 2.78 \text{ mg}$$

At the right is part of the graph of the function $y = \left(1 + \frac{1}{x}\right)^x$



One of the graph's asymptotes is $y = e$, where e is an irrational number approximately equal to 2.71828 **$e \approx 2.71828$**

Exponential functions with a base of e are useful for describing *continuous* growth or decay.

Put $y = \epsilon^x$ in your graphing calculator. Evaluate e^4, e^{-3}, $e^{½}$

54.598

0.4979

1.6487

The more frequently interest is compounded, the more quickly the amount in the account increases.

If interest is **compounded continuously**, it is easier to use the number ϵ.

$$A = Pe^{rt}$$

A = amount in account

P = Principal

r = annual rate of interest

t = time in years

Suppose you invest $1300 at an annual interest rate of 4.3% compounded continuously. Find the amount you will have in the account after three years.

$A = 1300e^{.043(3)}$

$A = \$1479$

Suppose you invest $100 at an annual interest rate of 4.8% compounded continuously. How much will you have in the account after three years? After 20 years?

$A = 100e^{.048(3)} = \$115.49$

$A = 100e^{.048(20)} = \$261.17$

homework:

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