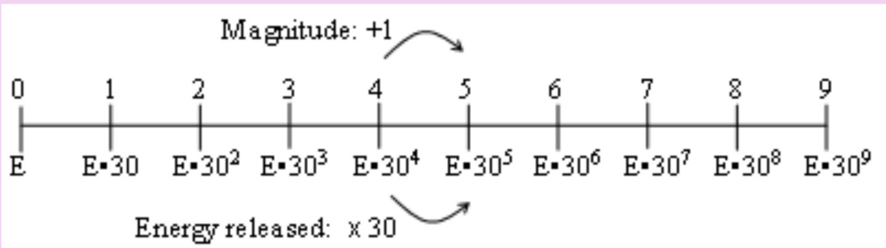
8.3 Logarithmic Functions as Inverses

Richter Scale – An exponential measure of earthquake magnitude. The magnitude of an earthquake is a measure of the amount of energy released at its source. An earthquake of magnitude 5 releases about 30 times as much energy as an earthquake of magnitude 4.



In 1997, an earthquake in Alabama registered 4.9 on the Richter scale. In 1999, one in California registered 7.0. Compare the energy released in the two quakes.

$$\frac{E \cdot 30^{7}}{E \cdot 30^{4.9}} = 30^{2.1} = 1265 \text{ times more}$$

Compare the amount of energy released in an earthquake that registered 6 on the Richter scale with one that registers 3.

$$\frac{E \cdot 30^{6}}{E \cdot 30^{3}} = 30^{3} = 27000 \text{ times more energy}$$

Logarithm: If $y = b^x$, then $\log_b y = x$

The exponent x in the exponential expression b^x is the logarithm in the equation $\log_b y = x$. The base b in b^x is the same as the base b in the logarithm. In both cases, $b \neq 1$ and $b > 0$.

A positive number b raised to any power x cannot equal a number y less than or equal to zero.

Therefore, the logarithm of a negative number or zero is undefined.

Write in logarithmic form:

$25 = 5^2$ — $\log_5 25 = 2$

$729 = 3^6$ — $\log_3 729 = 6$

$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ — $\log_{\frac{1}{2}}\left(\frac{1}{8}\right) = 3$

$10^0 = 1$ — $\log_{10} 1 = 0$

To evaluate logarightms, you can write them in exponential form.

Evaluate. (Write both with the same bases.)

$\log_8 16$

$16 = 8^x$

$2^4 = (2^3)^x$

$4 = 3x$

$4/3 = x$

$\log_9 27$

$27 = 9^x$

$3^3 = (3^2)^x$

$3 = 2x$

$3/2 = x$

$\log_{10} 100$

$100 = 10^x$

$10^2 = 10^x$

$2 = x$

$\log_{64} \frac{1}{32}$

$\frac{1}{32} = 64^x$

$2^{-5} = 2^{6x}$

$-5 = 6x$

$-5/6 = x$

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Common Logarithm: a logarithm that uses base 10. $\log_{10} y = \log y$

Scientists use common logarithms to measure acidity, which increases as the concentration of hydrogen ions in a substance increases. The pH of a substance equals $-\log[H^+]$, where $[H^+]$ is the concentration of hydrogen ions.

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Find the concentration of hydrogen ions in seawater of pH 8.5.

$-\log(8.5)$

$-.929$

Find the concentration of hydrogen ions in an apple of pH 3.3.

$-.519$

Find the concentration of hydrogen ions in a banana of pH 5.2. $-.716$

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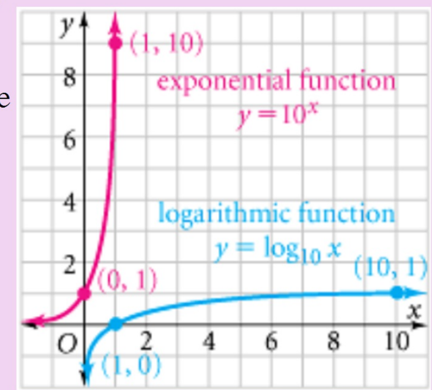
Logarithmic Function: The inverse of an exponential function.

You can graph $y = \log_b x$ as the inverse of $y = b^x$



The x-axis is the asymptote for the exponential function. The y-axis is the asymptote for the logarithmic function.

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Graph $y = \log_3 x$ $\quad y = \log_3(x+3) - 2$

make a table of $y = 3^x$ then switch the x and y coordinates to get the inverse (the logarithm)

x	y
-2	1/9
-1	1/3
0	1
1	3
2	9

$\log_3 x$

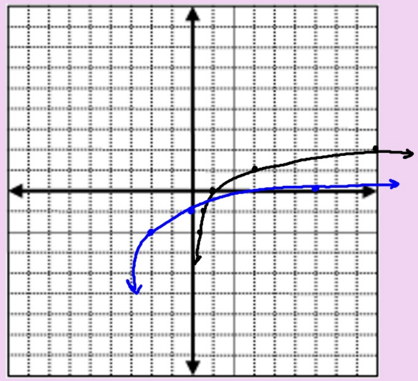
(1/9, -2)

$(\frac{1}{3}, -1)$

(1, 0)

(3, 1)

(9, 2)



homework:

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