8.5 Exponential and Logarithmic Equations

Exponential Equation: An equation of the form $b^{mn} = a$ where the exponent includes a variable.

If m and n are both positive and m ≠ n, then

You can therefore solve an exponential equation by taking the logarithm of each side of the equation.

Solving an Exponential Equation

$3^x = 4$

$x\frac{\log 3}{\log 3} = \frac{\log 4}{\log 3}$

$x = 1.2619$

$6^{2x} = 21$

$2x \log 6 = \log 21$

$2x = \frac{\log 21}{\log 6}$

$2x = 1.6992$

$x = .8496$

$3^{x+4} = 101$

$(x+4)\frac{\log 3}{\log 3} = \frac{\log 101}{\log 3}$

$x + 4 = 4.2009$

$x = 0.2009$

$3 - 5^{2x} = -13$

$-3 \quad -3$

$-5^{2x} = -16$

$\frac{-5^{2x}}{-1} = \frac{-16}{-1}$

$5^{2x} = 16$

$2x \log 5 = \log 16$

$2x = \frac{\log 16}{\log 5}$

$2x = 1.7227$

$x = .8614$

Change of Base Formula: For any positive numbers M, b, and c with b ≠ 1 and c ≠ 1.

$$\log_b M = \frac{\log_c M}{\log_c b}$$

Use the change of base formula to evaluate the following. Then convert to a logarithm in base 2.

$\log_3 15$

$\frac{\log 15}{\log 3} = 2.465$

$\log_3 15 = \log_2 x$

$2.465 = \log_2 x$

$\log 2(2.465 = \frac{\log x}{\log 2})$

$.7420 = \log x$

$10^{.7420} = 10^{\log x}$

$10^{.7420} = x$

$x = 5.5208$

$\log_6 12$

$\frac{\log 12}{\log 6} = 1.3869$

$\log_6 12 = \log_2 x$

$1.3869 = \frac{\log x}{\log 2}$

$.4175 = \log x$

$10^{.4175} = x = 2.6151$

You can use the Change of Base Formula to solve an exponential equation. Take the logarithm of each side using the base of the exponent as the base for the logarithm. Then use the Change of Base Formula.

$2^{3x} = 172$

$\log_2 2^{3x} = \log_2 172$

$3x = \frac{\log 172}{\log 2}$

$\frac{3x}{3} = \frac{7.426}{3}$

$x = 2.4754$

$7^{5x} = 3000$

$\log_7 7^{5x} = \log_7 3000$

$5x = \log_7 3000$

$5x = \frac{\log 3000}{\log 7}$

$5x = 4.1145$

$x = .8229$

$$\frac{15000}{5000} = \frac{5000}{5000}\left(1 + \frac{.1}{12}\right)^{12x}$$

$$3 = (1.0083)^{12x}$$

$$\log_{1.0083} 3 = \log_{1.0083} 1.0083^{12x}$$

$$\log_{1.0083} 3 = 12x$$

$$\frac{\log 3}{\log 1.0083} = 12x$$

Logarithmic Equation: An equation that includes a logarithmic expression.

In some cases, you need to use the properties of logarithms to simplify expressions before solving the equation.

Solve each equation.

$\log 5x + 3 = 6$

$-3 \quad -3$

$\log 5x = 3$

$10^{\log_{10} 5x} = 10^3$

$5x = 10^3$

$5x = 1000$

$x = 200$

$\log(7 - 2x) = -1$

$7 - 2x = 10^{-1}$

$7 - 2x = .1$

$-7 \qquad -7$

$\frac{-2x}{-2} = \frac{-6.9}{-2}$

$x = 3.45$

Solve each equation.

$\log(2x - 2) = 4$

$2x - 2 = 10^4$

$2x - 2 = 10{,}000$

$\frac{2x}{2} = \frac{10{,}002}{2}$

$x = 5{,}001$

$2\log x - 2 = 4$

$\log x^2 - 2 = 4$

$+2 \quad +2$

$\log x^2 = 6$

$x^2 = 10^6$

$\sqrt{x^2} = \sqrt{1{,}000{,}000}$

$x = 1{,}000$

Solve each equation.

$\log x + \log 2 = 5$

$\log 2x = 5$

$2x = 10^5$

$2x = 100{,}000$

$x = 50{,}000$

$\log 6 - \log 3x = -2$

$\log \frac{6}{3x} = -2$

$\frac{6}{3x} = 10^{-2}$

$\frac{6}{3x} = .01$

$\frac{2}{x} = .01$

$\frac{2}{.01} = \frac{.01x}{.01}$

$200 = x$

Solve each equation.

$3\log x - \log 2 = 5$

$\log \frac{x^3}{2} = 5$

$\frac{x^3}{2} = 10^5$

$\frac{x^3}{2} = 100000$

$x^3 = 200000$

$\sqrt[3]{x^3} = \sqrt[3]{200000}$

$x = 58.48$

$2\log x - \log 3 = 2$

$\log \frac{x^2}{3} = 2$

$\frac{x^2}{3} = 10^2$

$\frac{x^3}{3} = 100$

$x^3 = 300$

$\sqrt[3]{x^3} = \sqrt[3]{300}$

$x = 17.32$

homework:

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