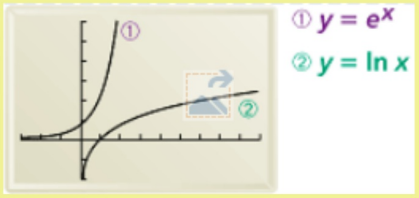
8.6 Natural Logarithms

Natural Logarithmic Function: If $y = e^x$ then $\log_e y = x$ which is commonly written as $\ln y = x$ (the inverse of $y = e^x$).



The properties of common logarithms apply to natural logarithms as well.

Write each expression as a single natural logarithm.

$5\ln 2 - \ln 4$

$\ln \frac{2^5}{4} = \ln \frac{32}{4}$

$= \ln 8$

$3\ln x + \ln y$

$\ln x^3 y$

$\frac{1}{4}\ln 3 + \frac{1}{4}\ln x$

$\ln 3^{1/4} x^{1/4}$

$\ln (3x)^{1/4}$

$\ln \sqrt[4]{3x}$

You can use the properties of logarithms to solve natural logarithmic equations.

Solve each equation. Check you answers.

$\ln x = 0.1$

$e^{\ln x} = e^{0.1}$

$x = e^{0.1}$

$x = 1.1052$

$\ln(3x - 9) = 21$

$e^{\ln(3x-9)} = e^{21}$

$3x - 9 = 1318815734$

$\frac{3}{3}x = \frac{1318815743}{3}$

$x = 439605247.8$

$\ln\left(\frac{x+2}{3}\right) = 12$

$\frac{x+2}{3} = e^{12}$

$\frac{x+2}{3} = 162755$

$x + 2 = 488264$

$-2 \quad -2$

$x = 488,262$

You can use natural logarithms to solve exponential equations.

Remember: If $y = e^x$, then $\ln y = x$ and $\ln e = 1$.

Use natural logarithms to solve each equation.

$e^{x+1} = 30$

$\ln e^{x+1} = \ln 30$

$x + 1 = \ln 30$

$x + 1 = 3.4012$

$-1 \quad -1$

$x = 2.4012$

$e^{\frac{2x}{5}} + 7.2 = 9.1$

$-7.2 \quad -7.2$

$e^{\frac{2x}{5}} = 1.9$

$\ln e^{\frac{2x}{5}} = \ln 1.9$

$\frac{2x}{5} = .6419$

$x = 1.6046$

$4e^{3x} + 1.2 = 14$

$-1.2 \quad -1.2$

$\frac{4e^{3x}}{4} = \frac{12.8}{4}$

$e^{3x} = 3.2$

$3x = \ln 3.2$

$3x = 1.1632$

$x = .3877$

An initial investment of $200 is worth $315.24 after seven years of continuous compounding. Find the interest rate.

$A = Pe^{rt}$

$\frac{315.24}{200} = \frac{200e^{r(7)}}{200}$

$1.5726 = e^{7r}$

$\ln 1.5762 = \ln e^{7r}$

$\frac{.455}{7} = \frac{7r}{7}$

$.065 = r$

6.5%

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An initial investment of $200 is now valued at $254.25. The interest rate is 6%, compounded continuously. How long has the money been invested?

$254.25 = 200e^{.06t}$

$1.27125 = e^{.06t}$

$\ln 1.27125 = .06t$

$\frac{.24}{.06} = \frac{.06t}{.06}$

$4 = t$

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A spacecraft can attain a stable orbit 300 km above Earth if it reaches a velocity of 7.7 km/s. The formula for a rocket's maximum velocity v in kilometers per second is $v = -0.0098t + c \ln K$. The booster rocket fires for t seconds and the velocity of the exhaust is c km/s. The ratio of the mass of the rocket with fuel to its mass without fuel is K.

Suppose a rocket used to propel a spacecraft has a mass ratio of about 15, an exhaust velocity of 2.1 km/s, and a firing time of 30s. Find the velocity of the spacecraft. Can the spacecraft achieve a stable orbit 300 km above Earth?

R, c, t

$v = -0.0098(30) + (2.1)\ln 15$

$-.294 + 5.687$

$v = 5.393$ km/s

no

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Find the velocity of a spacecraft whose booster rocket has a mass ratio of 22, an exhaust velocity of 2.3 km/s, and a firing time of 50 s. Can the spacecraft achieve a stable orbit 300 km above Earth?

$v = -0.0098(50) + 2.3 \ln 22$

$v = 6.62$

no

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homework:

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