

9.4 Rational Expressions

Simplify. State any restrictions on the variables.

$$\frac{-27x^3y}{9x^4y} = \frac{(-1)(3)(\cancel{9})(\cancel{x})(\cancel{x})(\cancel{x})(y)}{(1)(\cancel{9})(\cancel{x})(\cancel{x})(\cancel{x})(x)(y)} = \frac{-3}{x}$$

*only common factors may cancel (divide to =1).
Terms do not cancel.
Think (-) cancel!

$x \neq 0$
 $y \neq 0$ } original denominator can't equal zero.

Simplify. State any restrictions on the variables.

$$\frac{6-3x}{x^2-6x+8} \rightarrow \text{most our factors have } +x \text{ and decreasing exponents.}$$

commutative property: $-3x+6$
distributive property: $-3(x-2)$ } these 2 steps accomplish this goal.

$$\text{factors: } \frac{-3(\cancel{x-2})}{(\cancel{x-2})(x-4)} = \frac{-3}{x-4} \quad \begin{matrix} x \neq 2 \\ x \neq 4 \end{matrix}$$

Simplify. State any restrictions on the variables.

$$\frac{2x^2-3x-2}{x^2-5x+6} = \frac{-4 \mid -3}{4(1) \mid 1-4} \quad \begin{matrix} x & -2 \\ 2x & \boxed{2x^2 \mid -4x} \\ 1 & \boxed{x \mid -2} \end{matrix} \quad \frac{(2x+1)(\cancel{x-2})}{(x-3)(\cancel{x-2})}$$

$$= \frac{2x+1}{x-3}$$

$x \neq 3 \quad x \neq 2$

Simplify. State any restrictions on the variables.

$$\frac{x^2-6x-16}{x^2+5x+6} = \frac{(x-8)(\cancel{x+2})}{(\cancel{x+2})(x+3)} = \frac{x-8}{x+3}$$

$x \neq -2$
 $x \neq -3$

Simplify. State any restrictions on the variables.

$$\frac{54x^3y^{-1}}{3x^{-2}y} = \frac{54x^3x^2}{3y y^1} = \frac{54x^5}{3y^2} = \frac{(1)(3)(18)x^5}{(1)(3)y^2}$$

$$= \frac{18x^5}{y^2}$$

$$y \neq 0$$

* neg. exponents
simplify by
using reciprocal

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Simplify. State any restrictions on the variables.

$$\frac{x-8}{x^2-64} = \frac{\cancel{x-8}}{(x+8)\cancel{(x-8)}} = \frac{1}{x+8}$$

$$x \neq -8$$

$$x \neq 8$$

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Multiply. State any restrictions on the variables.

$$\frac{6x^2 \cdot 8y}{4y^3 \cdot 3x} = \frac{\cancel{2} \cancel{3} \cancel{x} \cancel{x}}{\cancel{2} \cancel{2} \cancel{y} \cancel{y} \cancel{y}} \cdot \frac{\cancel{2} \cancel{4} \cancel{y}}{\cancel{1} \cancel{3} \cancel{x}} = \frac{4x}{y^2}$$

$$x \neq 0$$

$$y \neq 0$$

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Multiply. State any restrictions on the variables.

$$\frac{a^2-4}{a^2-1} \cdot \frac{a+1}{a^2+2a}$$

$$\frac{\cancel{(a+2)} \cancel{(a-2)}}{\cancel{(a+1)} \cancel{(a-1)}} \cdot \frac{\cancel{a+1}}{\cancel{a(a+2)}} = \frac{a-2}{a(a-1)}$$

$$a \neq -1$$

$$a \neq 1$$

$$a \neq 0$$

$$a \neq -2$$

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Multiply. State any restrictions on the variables.

$$\frac{2x^2+7x+3}{x-4} \cdot \frac{x^2-16}{x^2+8x+15}$$

$\begin{array}{r|l} \cdot 6 & +7 \\ \hline 6(1) & 6+1 \end{array}$
 $\begin{array}{cc} & x & 3 \\ \hline 2x & 2x^2 & 6x \\ 1 & x & 3 \end{array}$
 $\frac{(2x+1)(x+3)}{(x-4)}$
 $\cdot \frac{(x+4)(x-4)}{(x+5)(x+3)}$

$$= \frac{(2x+1)(x+4)}{x+5} = \frac{2x^2+9x+4}{x+5}$$

$x \neq 4$ $x \neq -5$ $x \neq -3$

Divide. State any restrictions on the variables.

$$\frac{4-x}{10x-2} \div \frac{5x-20}{15x-3}$$

$$\frac{4-x}{10x-2} \cdot \frac{15x-3}{5x-20} \left. \vphantom{\frac{4-x}{10x-2} \cdot \frac{15x-3}{5x-20}} \right\} \text{multiply by reciprocal}$$

$$\frac{-1(x-4)}{2(5x-1)} \cdot \frac{3(5x-1)}{5(x-4)} = \frac{-3}{10}$$

$x \neq \frac{1}{5}$

Divide. State any restrictions on the variables.

$$\frac{a^2+2a-15}{a^2-16} \div \frac{a+1}{3a-12}$$

$$\frac{a^2+2a-15}{a^2-16} \cdot \frac{3a-12}{a+1}$$

$$\frac{(a+5)(a-3)}{(a+4)(a-4)} \cdot \frac{3(a-4)}{a+1} = \frac{3(a+5)(a-3)}{(a+4)(a+1)} = \frac{3a^2+6a-45}{(a+4)(a+1)}$$

$a \neq 4$ $a \neq -4$ $a \neq -1$

Divide. State any restrictions on the variables.

$$\frac{3-y}{(2x-1)(x+5)} \div \frac{6(y-3)}{(2x-1)(x-7)}$$

$$\frac{3-y}{(2x-1)(x+5)} \cdot \frac{(2x-1)(x-7)}{6(y-3)}$$

$$\frac{-1(y-3)}{(2x-1)(x+5)} \cdot \frac{(2x-1)(x-7)}{6(y-3)} = \frac{-(x-7)}{6(x+5)} = \frac{-x+7}{6x+30}$$

$x \neq \frac{1}{2}$ $x \neq -5$ $x \neq 7$

Divide. State any restrictions on the variables.

$$\frac{3x^2+5x-2}{x-5} \div \frac{3x^2-7x+2}{x^2-25}$$

$$\frac{3x^2+5x-2}{x-5} \cdot \frac{x^2-25}{3x^2-7x+2}$$

$$\frac{\begin{array}{c|c} -6 & +5 \\ \hline 6(-1) & 6(-1) \end{array} \quad \begin{array}{c} x \quad 2 \\ 3x \begin{array}{|c|c|} \hline 3x^2 & 6x \\ \hline -x & -2 \\ \hline \end{array} \\ \hline -1 \begin{array}{|c|c|} \hline -x & -2 \\ \hline \end{array} \end{array}}{(3x-1)(x+2)} \cdot \frac{(x+5)(x-5)}{(x-5)(3x-1)(x-2)}$$

$$= \frac{(x+2)(x+5)}{x-2} \quad x \neq 5 \quad x \neq -5$$