

## 9.6 Solving Rational Equations

Extraneous solutions can be introduced when you multiply both sides of an equation by the same algebraic expression. An extraneous solution is a solution derived from the equation but does not make the equation true.

You must check all solutions to find out if any are extraneous.

$$\frac{1}{9} = \frac{x}{18} \quad 2 \cdot 18 \cdot \frac{1}{9} = \frac{x}{18} \cdot 18$$

$$1 \cdot 18 = 9 \cdot x$$

$$18 = 9x$$

$$2 = x$$

Page 1

Solve. Check each solution.

cross products

$$\frac{-2}{x^2-2} \cdot \frac{2}{x-4}$$

$$-2(x-4) = 2(x^2-2)$$

$$-2x + 8 = 2x^2 - 4$$

$$+2x \quad -8 \quad +2x \quad -4$$

$$0 = 2x^2 + 2x - 12$$

$$\frac{0}{2} = \frac{2x^2 + 2x - 12}{2}$$

$$0 = x^2 + x - 6$$

$$0 = (x+3)(x-2)$$

$$x = -3 \quad x = 2$$

$$\frac{-2}{(-3)^2-2} = \frac{2}{-3-4} \quad \frac{-2}{2^2-2} = \frac{2}{2-4}$$

$$\frac{-2}{9-2} = \frac{2}{-7} \quad \frac{-2}{4-2} = \frac{2}{-2}$$

$$\frac{-2}{7} = \frac{2}{-7} \quad \frac{-2}{2} = \frac{2}{-2}$$

$$\frac{-2}{7} \neq \frac{2}{-7} \quad \frac{-2}{2} \neq \frac{2}{-2}$$

$$-1 \neq -1$$

Page 2

Solve. Check each solution.

$$\frac{1}{x-3} = \frac{6x}{x^2-9}$$

$$\frac{1}{x-3} = \frac{6x}{(x-3)(x+3)}$$

$$\cancel{(x-3)}(x+3) \cdot \frac{1}{\cancel{(x-3)}} = \cancel{(x-3)}(x+3) \cdot \frac{6x}{\cancel{(x-3)}(x+3)}$$

$$x+3 = 6x$$

$$-x \quad -x$$

$$\frac{3}{5} = \frac{5x}{5}$$

$$\frac{3}{5} = x$$

$$\frac{1}{\frac{3}{5}-3} = \frac{6(\frac{3}{5})}{(\frac{3}{5})^2-9}$$

$$\frac{1}{-2.4} = \frac{18}{\frac{9}{25}-9}$$

$$\frac{1}{-2.4} = \frac{18}{\frac{3.6}{25}-9}$$

$$\frac{1}{-2.4} \neq \frac{18}{-8.64}$$

$$-0.41\bar{6} \neq -0.417$$

Page 3

Solve. Check each solution.

$$\frac{-4}{5(x+2)} = \frac{3}{x+2}$$

$$\cancel{5(x+2)}(-4) = \cancel{5(x+2)} \cdot \frac{3}{\cancel{(x+2)}}$$

$$-4 \neq 15$$

no solution

$$-4(x+2) = 3 \cdot 5(x+2)$$

$$-4x - 8 = 15x + 30$$

$$-15x \quad -15x$$

$$-19x - 8 = 30$$

$$-19x = 38$$

$$x = -2$$

Page 4

Solve. Check each solution.

$$\frac{3 \cdot 3}{5x \cdot 3} - \frac{4 \cdot 5}{3x \cdot 5} = \frac{1}{3}$$

$$\frac{9}{15x} - \frac{20}{15x} = \frac{1}{3}$$

$$\frac{-11}{15x} = \frac{1}{3}$$

$$\frac{-33}{15} = \frac{15x}{15}$$

$$\frac{-11}{5} = x$$

$$\frac{3}{5(\frac{11}{5})} - \frac{4}{3(\frac{11}{5})} = \frac{1}{3}$$

$$-\frac{3}{11} - \frac{4}{-\frac{33}{5}} = \frac{1}{3}$$

$$-\frac{3}{11} + \frac{4}{6.6} = \frac{1}{3}$$

$$.3 = .3$$

Page 5

Solve. Check each solution.

$$\frac{4}{x} - \frac{3}{x+1} = 1$$

$$\frac{4(x+1)}{x(x+1)} - \frac{3(x)}{x+1(x)} = 1$$

$$\frac{4x+4}{x(x+1)} - \frac{3x}{x(x+1)} = 1$$

$$\frac{x+4}{x(x+1)} = 1$$

$$x+4 = x^2+x$$

$$-x-4 = -x-4$$

$$0 = x^2 - 4$$

$$x = -2 \quad x = 2$$

$$\frac{4}{-2} - \frac{3}{-2+1} = 1$$

$$-2 + 3 = 1$$

$$\frac{4}{2} - \frac{3}{2+1} = 1$$

$$2 - \frac{3}{3} = 1$$

$$2 - 1 = 1$$

$$1 = 1$$

Page 6

Solve. Check each solution.

$$\frac{1 \cdot 5}{2x \cdot 5} - \frac{2 \cdot 2}{5x \cdot 2} = \frac{1}{2}$$

$$\frac{5}{10x} - \frac{4}{10x} = \frac{1}{2}$$

$$\frac{1}{10x} = \frac{1}{2}$$

$$2 = 10x$$

$$\frac{1}{5} = x$$

$$\frac{1}{2(\frac{1}{5})} - \frac{2}{5(\frac{1}{5})} = \frac{1}{2}$$

$$\frac{1}{\frac{2}{5}} - \frac{2}{1} = \frac{1}{2}$$

$$\frac{5}{2} - 2 = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

Page 7

Rosa can jog 5 miles downhill in the same time it takes her to jog 3 miles uphill. She jogs downhill 4 mph faster than she jogs uphill. Find her jogging rate each way.

down  
uphill

$$\frac{5}{3} = \frac{4+x}{x}$$

$$\frac{10}{6} \quad \text{downhill}$$

$$\text{uphill}$$

$$5x = 3(4+x)$$

$$5x = 12 + 3x$$

$$2x = 12$$

$$x = 6$$

Page 8

Jim and Alberto have to paint 6000 square feet of hallway in an office building. Alberto works twice as fast as Jim. Working together, they can complete the job in 15 hours. How long would it take each of them working alone?

	time	rate
Jim	$2x$	$\frac{6000}{2x}$
Alberto	$x$	$\frac{6000}{x}$
Combined	15	$\frac{6000}{15}$

$J + A = \text{combined}$   
 $\frac{6000}{2x} + \frac{6000}{x} = \frac{6000}{15}$   
 $\frac{3000}{x} + \frac{6000}{x} = 400$   
 $\frac{9000}{x} = 400$   
 $9000 = 400x$   
 $22.5 = x$

Alberto 22.5 hr  
 Jim 45 hrs  $\rightarrow (22.5 \cdot 2)$

Page 9

Suppose Adrian can weed the garden twice as fast as his son Phillip. Together they can weed the garden in 3 hours. How long would it take each of them working alone? faster = less time

	time	rate
Adrian	$x$	$\frac{1}{x}$
Phillip	$2x$	$\frac{1}{2x}$
combined	3	$\frac{1 \text{ garden}}{3 \text{ hours}}$

$A + P = c$   
 $\frac{1}{x} + \frac{1}{2x} = \frac{1}{3}$      $\frac{2}{2x} + \frac{1}{2x} = \frac{1}{3}$   
 $\frac{3}{2x} = \frac{1}{3}$

$\frac{3}{2x} = \frac{1}{3}$   
 $9 = 2x$   
 $4.5 = x$   
 4.5 hrs. Adrian  
 9 hrs. Phillip

*continued*

Page 10

homework:

page 515 # 1-25 odd