Quick Check:

If you can correctly answer the questions below, you are ready to move onto Activity #2.

1. Determine which of the following are rational functions. State why you chose your answer.

$p(x)=\frac{\sqrt{x^{1/2}}+2x+1}{x^2+2x-1}$ $g(x)=\frac{2x^3+x^2-1}{x+3}$ $h(x)=\frac{2^x+2}{x+2}$

2. Given $f(x)=\frac{6x^2-13x-5}{x^2-5x+4}$, find

$-30 \mid -13$

$-15(2) \mid 2-15$

	$2x$	-5
$3x$	$6x^2$	$-15x$
1	$2x$	-5

$\frac{(3x+1)(2x}{(x-4)(x-}$

1. The domain of $f(x)$.

   all real numbers except $x=4,\ x=1$

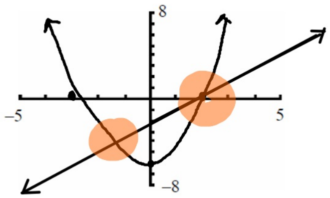
2. The roots of $f(x)$.

   $\left(-\frac{1}{3},0\right)\ \left(\frac{5}{2}\right)$

3. The vertical asymptotes, or infinite discontinuities, of $f(x)$.

   $x=4,\ x=1$

a) Use your graphing calculator to graph $f(x)=x^2+x-6$ and $g(x)=x-2$ on the same viewing window. As accurately as possible, sketch the graph of these functions on the given grid below. Where do the graphs intersect?



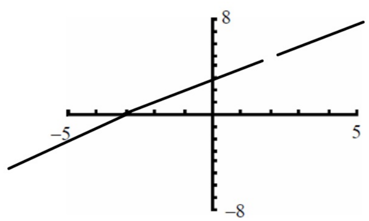
b) Let $h(x)=\frac{f(x)}{g(x)}=\frac{x^2+x-6}{x-2}$. Rewrite $h(x)$ with its numerator factored. $h(x)=\frac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}}$

- What do you notice about the factor $x-2$? in both num. & denom.
- What type of function results when $h(x)$ is simplified? linear
- What do you think the graph of $h(x)$ will look like? State specifically what you think the graph will look like at $x = 2$. Straight line; hole at $x=2$

c) Use your graphing calculator to graph $h(x)$. Use ZOOM → #4 decimal and then adjust the window settings so ymin = –6.2 and ymax = 6.2.

- Does the graph look different at $x = 2$? Describe what you see. yes, it looks like a hole.
- What does the table show at $x = 2$?

d) Give an accurate sketch of $h(x)$ on the grid below.



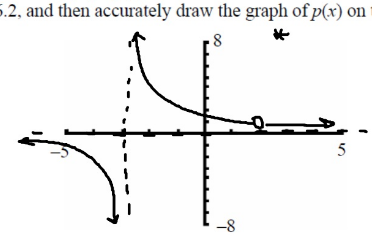
e) Without using your calculator, what do you think the graph of $p(x)=\frac{g(x)}{f(x)}$ would look like? List all intercepts, vertical asymptotes (infinite discontinuities), and holes (removable discontinuities).

VA: $x=-3$

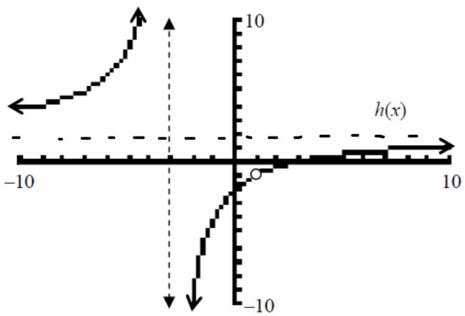
hole: $x=2$

$$\frac{\cancel{(x-2)}}{\cancel{(x-2)}(x+3)}=\frac{1}{x+3}$$

f) Use your graphing calculator to graph $p(x)$. Use ZOOM → #4 decimal and then adjust the window settings so ymin = –6.2 and ymax = 6.2, and then accurately draw the graph of $p(x)$ on the grid below.

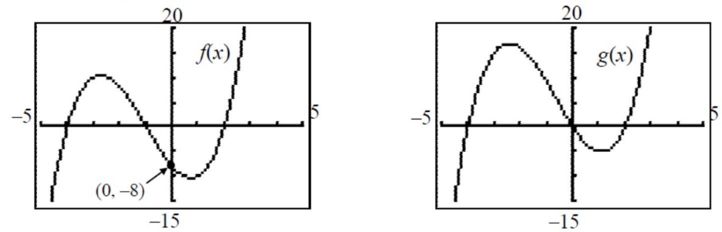


a) A graph of $h(x)$ is given below.



- Give the roots of the function. $(4,0)$
- Give the y-intercept of the function. $(0,-2)$
- Give the infinite discontinuities of the function. $x=-3$
- Give the removable discontinuities of the function. $x=1$
- Give a possible equation for $h(x)$. $\dfrac{2(x-1)(x-4)}{(x-1)(x+3)}$

Graphs of $f(x)$ and $g(x)$ are given below.



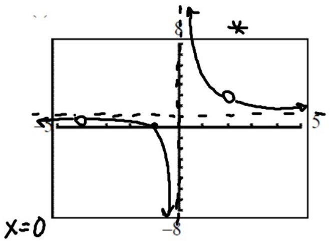
a) What are the roots of $f(x)$? $(-4,0)$ $(-1,0)$ $(2,0)$

b) What are the roots of $g(x)$? $(-4,0)$ $(0,0)$ $(2,0)$

c) Let $h(x)=\dfrac{f(x)}{g(x)}$. $\dfrac{(x+4)(x+1)(x-2)}{x(x+4)(x-2)}$

- What are the roots of $h(x)$? $x=-1$
- Does $h(x)$ have any infinite discontinuities (vertical asymptotes)? $x=0$
- Does $h(x)$ have any removable discontinuities (holes)? $x=-4$ $x=2$
- Without using your calculator, sketch a graph of $h(x)$. (Use the grid below.) zero $(-1,0)$
- Give a possible equation for $h(x)$.

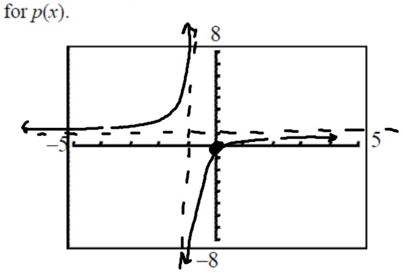
HA = 1



d) Let $p(x)=\dfrac{g(x)}{f(x)}$. $\dfrac{x(x+4)(x-2)}{(x+4)(x+1)(x-2)}$

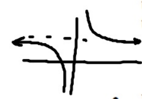
- What are the roots of $p(x)$? $(0,0)$
- Does $p(x)$ have any infinite discontinuities (vertical asymptotes)? $x=-1$
- Does $p(x)$ have any removable discontinuities (holes)? $x=-4$ $x=2$
- Without using your calculator, sketch a graph of $p(x)$. (Use the grid below.)
- Give a possible equation for $p(x)$.

HA: $y=1$



a) Use your graphing calculator to graph $f(x)=\dfrac{3x+1}{x+1}$.

- Go to your table (2nd GRAPH), scroll down and examine the $f(x)$ values as the x values tend toward positive infinity. What do you see? How would you use mathematical notation (symbols) to write this?
  $f(x)\to 3$
- Go to your table, scroll up and examine the $f(x)$ values as the x values tend toward negative infinity. What do you see? How would you use mathematical notation (symbols) to write this?
  $f(x)\to 3$



- Press the TRACE button on your calculator and use the arrow buttons to trace the graph as the x values tend toward positive infinity. What is happening to the function's values as you trace along the graph? Is this matching up with what you found above?
  $f(x)\to 3$
- Press the TRACE button on your calculator and use the arrow buttons to trace the graph as the x values tend toward negative infinity. What is happening to the function's values as you trace along the graph? Is this matching up with what you found above?
  $f(x)\to 3$
- Look at the equation for $f(x)$. Can you explain why the function's values get closer and closer to the value found above as the x values tends toward positive and negative infinity?
  $y=3$
- Do the $f(x)$ values have this same value for small x values ($x=-5,-4,-3,-2,\ldots..3,4,5$)? Why or why not? no. on opposite sides
- What is the equation for the horizontal asymptote of $f(x)$ above?

b) Use your graphing calculator to graph $h(x) = \frac{4x^2+1}{-2x^2-3}$.

- Go to your table, scroll down and examine the $h(x)$ values as the x values tend toward positive infinity. What do you see? How would you use mathematical notation (symbols) to write this?

  $f(x) \to -2$

- Go to your table, scroll up and examine the $h(x)$ values as the x values tend toward negative infinity. What do you see? How would you use mathematical notation (symbols) to write this?

  $f(x) \to -2$

- Press the TRACE button on your calculator and use the arrow buttons to trace the graph as the x values tend toward positive infinity. What is happening to the function's values as you trace along the graph? Is this matching up with what you found above?

  $f(x) \to -2$

- Press the TRACE button on your calculator and use the arrow buttons to trace the graph as the x values tend toward negative infinity. What is happening to the function's values as you trace along the graph? Is this matching up with what you found above?

  $f(x) \to -2$

- What is the equation for the horizontal asymptote of $h(x)$ above?

  $y = -2$

- Look at the equation for $h(x)$, can you explain why the function's values get closer and closer to the value found above as the x values tends toward positive and negative infinity?

  b/c it's the asymptote

- Do the $h(x)$ values have this same value for small x values ($x = -5, -4, -3, -2, \ldots 3, 4, 5$)? Why or why not? no. on opposite sides

c) Use your graphing calculator to graph $p(x) = \frac{2x^3+4x^2-6x}{x^3+x^2-16x+20} = \frac{2x(x+3)(x-1)}{(x-2)^2(x+5)}$. Does $p(x)$ appear to have a horizontal asymptote? If so, give the equation of the asymptote?

yes. $y = 2$

d) Use your graphing calculator to graph $q(x) = \frac{x+2}{2x^2-2} = \frac{x+2}{2(x+1)(x-1)}$. Does $q(x)$ appear to have a horizontal asymptote? If so, give the equation of the asymptote?

yes. $y = 0$

e) Use your graphing calculator to graph $r(x) = \frac{2x^2+5x-3}{x^3+3x^2+2x}$. Does $q(x)$ appear to have a horizontal asymptote? If so, give the equation of the asymptote?

yes. $y = 0$

f) Use your graphing calculator to graph $s(x) = \frac{2x^3+x^2-1}{x^2}$. Does $q(x)$ appear to have a horizontal asymptote? If so, give the equation of the asymptote?

no.

g) Use your graphing calculator to graph $y(x) = \frac{-3x^3+x^2-1}{x^2-1}$. Does $q(x)$ appear to have a horizontal asymptote? If so, give the equation of the asymptote?

h) Record your finding in a-g above in the following table.

Function	Leading Term in Numerator	Leading Term in Denominator	Equation for Horizontal Asymptote
a) $f(x) = \frac{3x+1}{x+1}$	3	1	$y = 3$
b) $h(x) = \frac{4x^2+1}{-2x^2-3}$	4	-2	$y = -2$
c) $p(x) = \frac{2x^3+4x^2-6x}{x^3+x^2-16x+20} = \frac{2x(x+3)(x-1)}{(x-2)^2(x+5)}$	2	1	$y = 2$
d) $q(x) = \frac{x+2}{2x^2-2}$	1	2	$y = 0$
e) $r(x) = \frac{2x^2+5x-3}{x^3+3x^2+2x}$	2	1	$y = 0$
f) $s(x) = \frac{2x^3+x^2-1}{x^2}$	2	1	none
g) $y(x) = \frac{-3x^3+x^2-1}{x^2-1}$	-3	1	none

- Some patterns/relationships exist in the table above that will help you determine the horizontal asymptote of a rational function. Can you find them? a-c

- Some patterns/relationships exist in the table above that will help you determine the horizontal asymptote of a rational function. Can you find them?

  if degrees equal, y = ratio of leading terms
  if num. degree higher, no HA
  If denom. degree higher, y = 0

- Use the findings of the table and give an equation for of a rational function that satisfies the following.(do not use any of the rational functions given above)

  i) Has a horizontal asymptote at $y = -5$. equation: $y = \frac{-5(x+2)}{x-3}$

  ii) Has a horizontal asymptote at $y = 0$. equation: $y = \frac{1}{x-1}$

  iii) Does not have a horizontal asymptote. equation: $y = \frac{2x^3}{x+7}$

  iv) Has a root at $x = 4$, a hole at $x = -3$, a vertical asymptote at $x = -1$, and a horizontal asymptote at $y = 2$.

  equation: $y = \frac{2(x-4)(x+3)}{(x+1)(x+3)}$

answers may vary