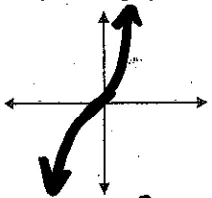
Notes

Polynomial Functions
Algebra 2: End Behavior

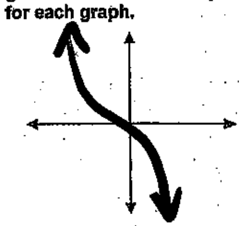
Name: ____________________

Graph each function using a graphing calculator. Be sure to choose a large window. Make a simple sketch of the general shape of the graph. Fill in the end behavior pattern for each graph.

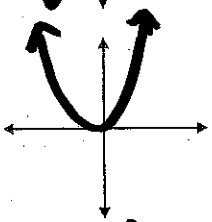
1. $f(x) = x^3$
(↙, ↗)



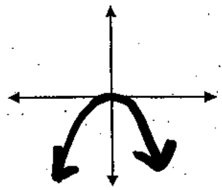
2. $f(x) = -x^3$
(↖, ↘)



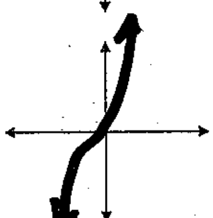
3. $f(x) = x^4$
(↖, ↗)



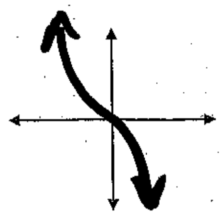
4. $f(x) = -x^4$
(↙, ↘)



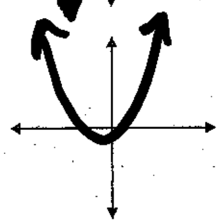
5. $f(x) = x^5$
(↙, ↗)



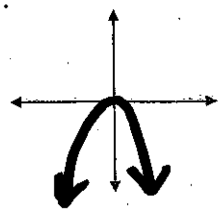
6. $f(x) = -x^5$
(↖, ↘)



7. $f(x) = x^6$
(↖, ↗)



8. $f(x) = -x^6$
(↙, ↘)



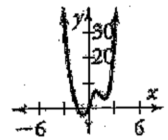
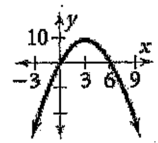
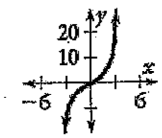
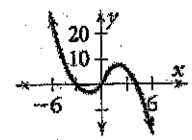
Make a conjecture about the end behavior for the general function, $f(x) = ax^n$.

1. **If the degree is even (n is even) the ends go in** same **direction(s).** (the same, opposite)
2. **If the degree is odd (n is odd) the ends go in** opposite **direction(s).** (the same, opposite)
3. **If the leading coefficient is positive (a > 0) the right end goes** up. (up, down)
4. **If the leading coefficient is negative (a < 0) the right end goes** down. (up, down)

You can determine *by inspection* the end behavior of the graph of a polynomial function in standard form. Look at the **coefficient** and the **degree** of the leading term.

RIGHT: If the leading coefficient is + **, then the graph rises to the right. If the leading coefficient is** – **, then the graph falls to the right.**

LEFT: If the degree of the polynomial is even **, then the left behavior is the SAME as the right behavior. If the degree of a polynomial is** odd **, then the left behavior is OPPOSITE of the right behavior.**

Up and Up	Down and Down	Down and Up	Up and Down
(↖, ↗)	(↙, ↘)	(↙, ↗)	(↖, ↘)
Example	Example	Example	Example
			
$y = x^4 - 3x^3 + 5x$	$y = -x^2 + 6x$	$y = x^3$	$y = -0.3x^3 + 4x + 2$
a > 0 and degree is even	a < 0 and degree is even	a > 0 and degree is odd	a < 0 and degree is odd

Exercises:

Complete the exercises without a calculator. Check your answers with a calculator. Determine the following: a) the leading coefficient, b) the degree of the polynomial, and c) the end behavior of the graph. Answers to part (c) will be in the following form:

(↖, ↗) (↙, ↘) (↙, ↗) (↖, ↘)

1. $y = 3x + 2$
(↙, ↗)

2. $y = 4x^3$
(↙, ↗)

3. $g(t) = -t^2 + t$
(↙, ↘)

4. $f(x) = 2x + x^5$
(↙, ↗)

5. $g(x) = -7x^8$
(↙, ↘)

6. $y = 3x^5 - 4x^4$
(↙, ↗)

Notes

Polynomial Functions Name: ______________________

Polynomial Functions and Their Graphs (Patterns of Polynomials)

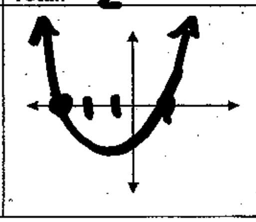
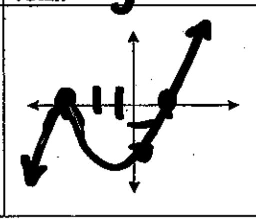
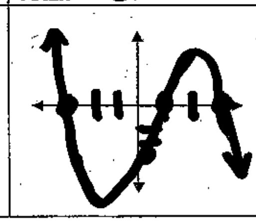
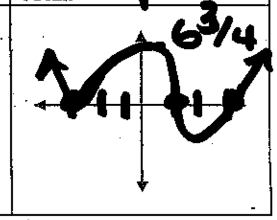
Fill out this table. Sketch the graph of this function using the critical points.

deg − 1

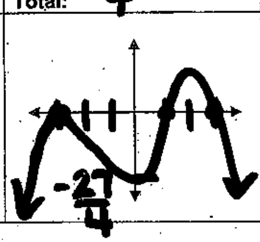
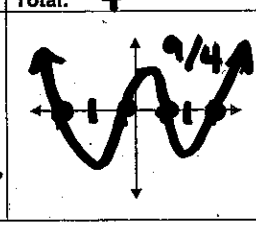
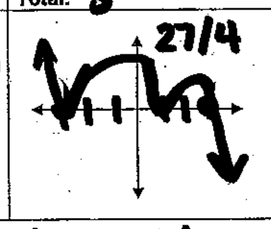
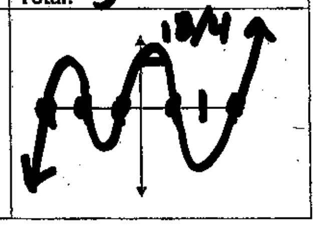
x = 0

cross-odd deg.

touch-even deg.

Function	$f(x) = \frac{1}{4}(x-1)^1(x+3)^1$	$f(x) = \frac{1}{4}(x-1)(x+3)^2$	$f(x) = -\frac{1}{4}(x-1)(x+3)(x-3)$	$f(x) = \frac{1}{4}(x-1)(x+3)^2(x-3)$
Leading Coefficient "a"	1/4	1/4	−1/4	1/4
Degree (count factors)	2 *	3	3	4
Number of Linear Factors (same as degree)	2 *	3	3	4
End Behavior (use arrows)	(↑, ↑)	(↓, ↑)	(↑, ↓)	(↑, ↑)
Number of Turning Points	2 − 1 = 1	3 − 1 = 2	3 − 1 = 2	4 − 1 = 3
y-intercept	−3/4	−9/4	−9/4	27/4 = 6 3/4
Number of Real Zeros	Crossing: 2 (1,0) (−3,0) Touching: 0 Total: 2 *	Crossing: 1 (1,0) Touching: 2 (−3,0) Total: 3	Crossing: 3 (1,0)(−3,0)(3,0) Touching: 0 Total: 3	Crossing: 2 (1,0)(3,0) Touching: 2 (−3,0) Total: 4
Sketch of Graph put dots on zeros and y-intercept				

Source: Claudia Heinrich and Kimberly Meyer-2008 DACTM Conference

Function	$f(x) = -\frac{1}{4}(x-1)(x+3)^2(x-3)$	$f(x) = \frac{1}{4}(x+1)(x-1)(x+3)(x-3)$	$f(x) = -\frac{1}{4}(x-1)^2(x+3)^2(x-3)$	$f(x) = \frac{1}{4}(x+1)(x-1)(x+2)(x-3)(x+3)$
Leading Coefficient	−1/4	1/4	−1/4	1/4
Degree	4	4	5	5
Number of Linear Factors	4	4	5	5
End Behavior	(↓, ↓)	(↑, ↑)	(↑, ↓)	(↓, ↑)
Number of Turning Points	3	3	4	4
y-intercept	−27/4	9/4	27/4	18/4
Number of Real Zeros	Crossing: 2 (1,0)(3,0) Touching: 2 (−3,0) Total: 4	Crossing: 4 (3,0) (−1,0)(1,0)(−3,0) Touching: 0 Total: 4	Crossing: 1 (3,0) Touching: 4 (1,0)(−3,0) Total: 5	Crossing: 5 Touching: 0 Total: 5
Sketch of Graph				

The maximum possible number of turning points is ___one less than___ the degree of the polynomial.

The maximum possible number of zeros of a polynomial is ___the same as___ its degree.

Touching zeros have ___even___ multiplicity. Crossing zeros have ___odd___ multiplicity.