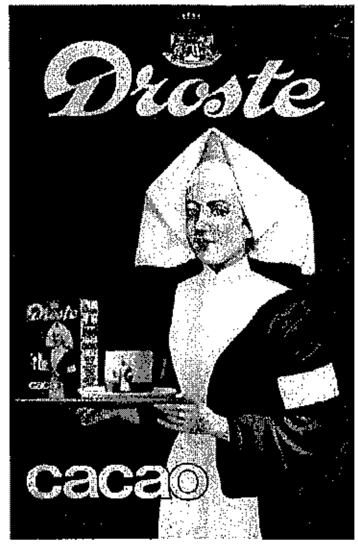
Recursive Sequences

Name____________________________

The recursive picture, seen at the right, is referred to as the Droste effect. The woman in the picture is holding an object which contains a smaller picture of her holding the same object, which, in turn, contains a smaller picture of her holding the same object, and so on and so on.



14 1   11 7 8 4 10 12 14 5 7 8

10 4 6 11 10 12 3 1 7,   13 1 11

2 11 12 14   9 3 10 12 14

11 7 8 4 10 12 14 5 7 8   10 4 6 11 10 12 3 1 7.

Solve the problems and find the answers in the Answer Vault. Using the letters, decode the message.

1. Find the first four terms of the sequence: $a_1 = 2,\ \ a_n = a_{n-1} + 6$

2. Find the first four terms of the sequence: $a_1 = 2,\ \ a_n = (a_{n-1})^2 + 3$

3. Find the first four terms of the sequence: $a_1 = 2,\ \ a_n = (-1)^{n-1} \bullet 3a_{n-1}$

4. Find the first four terms of the sequence: $a_1 = 2,\ \ a_{n+1} = n \bullet a_n$

5. Find the first four terms of the sequence: $a_1 = 2,\ \ a_{n+1} = 3a_n + n$

6. Write a recursive formula for the sequence: 2, 4, 8, 16, …

7. Write a recursive formula for the sequence: 2, -6, 18, -54, …

8. Write a recursive formula for the sequence: 2, 4, 6, 8, …

9. Write a recursive formula for the sequence: 2, -4, -10, -16, ...

10. Write a recursive formula for the sequence: 2, 5, 26, 677, ...

11. One of the drawbacks to working with recursive sequences is that :
a) The sequences must always contain only positive values.
b) The computations are extremely difficult and require a calculator in nearly all cases.
c) The computation of a 100^{th} term requires the computation of every term preceding it.
d) There are no drawbacks to working with recursive sequences.

12. The recursive formula $a_1 = 3,\ a_n = 3a_{n-1}$ and the explicit formula $a_n = 3^n$:
a) represent the same sequence.
b) create sequences with the same values for terms one through ten, but differ after that point.
c) create sequences with the same values for terms one through forty, but differ after that point.
d) represent completely different sequences.

13. Which of the following is NOT an example of a recursive sequence:
a) 9, -18, 36, -72, ...
b) $3,\ 3\sqrt{3},\ 9,\ 9\sqrt{3},\ ...$
c) -2, -4, -8, -16, ...
d) 2, 3, 5, 7, 11, ...

14. The terms in the recursive sequence $a_1 = 1,\ a_{n+1} = \sqrt[2]{a_n} + 7$:
a) get continually smaller as the numbers of the terms increase.
b) get continually larger as the numbers of the terms increase.
c) vary between increasing and decreasing in value as the numbers of the terms increase.
d) have the same value after the 10^{th} term.

ANSWER VAULT:

A 2, 7, 23, 72	**C** $a_1 = 2$, $a_n = 2a_{n-1}$	**D** $a_1 = 2$, $a_n = a_{n-1} + 2$	**E** 2, 2, 4, 12	**F** $a_1 = 2$, $a_{n+1} = a_n - 6$
H 2, 4, 8, 16	**I** 2, -6, -18, 54	**M** 2, 7, 52, 2707	**N** $a_1 = 2$, $a_{n+1} = -3a_n$	**O** 2, 8, 14, 20
R $a_1 = 2$, $a_{n+1} = (a_n)^2 + 1$	**S** Choice a	**T** Choice b	**U** Choice c	**Y** Choice d