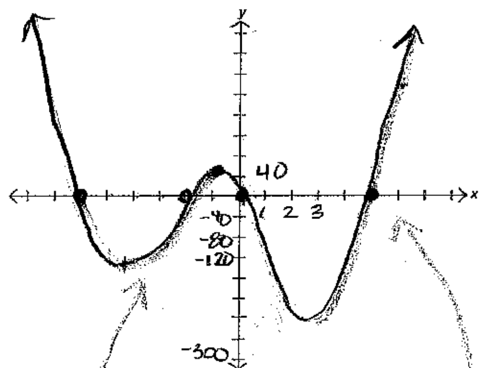
Behavior of Polynomials[1]

1. Given the polynomial $f(x) = x^4 + 3x^3 - 28x^2 - 60x$ answer the following questions:
   a. What is the degree of f?

      4, quartic

   b. Use your graphing calculator to draw a graph of f. Be sure to choose a window that allows you to see the whole graph.



   c. Identify all of the intercepts of f. Write your answers as points (a, b).

      (-6,0) (-2,0) (0,0) (5,0)

   d. Rewrite the polynomial in factored form.

      $y = x(x+6)(x+2)(x-5)$

   e. Identify where all the intervals where f is increasing and decreasing. Write your answers in interval notation.

      inc. [-4, -1] [3, ∞)
      dec. (-∞, -4] [-1, 3]

   f. Estimate the location and value of any relative extrema (e.g. local minimums and local maximums).

      min @ -4 value -160 min @ 3 value -275
      max @ -1 value 30

   g. Identify the end behavior of the graph.

      As $x \to +\infty$, $f(x) \to \infty$

      As $x \to -\infty$, $f(x) \to \infty$

[1] MAT150 College Algebra

Degrees and Zeros[2]

2. Determine the degree and zeros of the following polynomials

Polynomial	Degree	Zeros
$f(x) = (x+3)(x-5)^2$	3	(-3,0) (5,0)
$g(x) = (x+2)(x-1)(x-4)$	3	(-2,0) (1,0) (4,0)
$k(x) = (3x+2)(x-3)^2$	3	(-2/3,0) (3,0)
$h(x) = x-3$	1	(3,0)
$m(x) = x(x-3)$	2	(0,0) (3,0)
$n(x) = x^2(x-3)$	3	(0,0) (3,0)
$p(x) = x^2(x-3)^2$	4	(0,0) (3,0)
$q(x) = x^2(x-3)^3$	5	(0,0) (3,0)

3. Take a look at the graphs of the last five functions from the previous problem:

$h(x) = x-3$ $m(x) = x(x-3)$ $n(x) = x^2(x-3)$ $p(x) = x^2(x-3)^2$ $q(x) = x^2(x-3)^3$

What do you notice at the zeros? What kind of generalizations can you make?

When a zero is repeated an even number of times.......... it touches the x-axis. and stays on the same side

When a zero is repeated an odd number of times........ it crosses the x-axis.

[2] MAT150 College Algebra

[3]End Behavior

4. Describe the end behavior of the following polynomials. See if you can complete this activity without using your calculator.

a. $f(x) = x^7 - 3x^6 + 45x^4 - 321x + 729$

as $x \to \infty$, $f(x) \to \infty$
as $x \to -\infty$, $f(x) \to -\infty$

b. $g(x) = x^2 + 3x^4 - 17x^7 + 3x^3 + 17 - 4x$

as $x \to \infty$, $f(x) \to -\infty$
as $x \to -\infty$, $f(x) \to \infty$

c. $h(x) = x(x-2)(3x-1)(4x-2)(4x+2)^2$

as $x \to \infty$, $f(x) \to \infty$
as $x \to -\infty$, $f(x) \to \infty$

d. $k(x) = -2(x-3)^2(2x-5)(x^2+4)$

as $x \to \infty$, $f(x) \to -\infty$
as $x \to -\infty$, $f(x) \to \infty$

[3] MAT150 College Algebra

[4]Finding Polynomials

5. Find polynomials for the following situations. You may leave your answers in factored form.
   a. Only one root at 5

$y = x - 5$

   b. Roots at 3 and -6

$y = (x-3)(x+6)$

   c. Roots at -3, 2 and 7

$y = (x+3)(x-2)(x-7)$

   d. Roots at -3, 2, and 3 and goes through the point (4,7)

$y = a(x+3)(x-2)(x-3)$
$7 = a(4+3)(4-2)(4-3)$
$y = \frac{1}{2}(x+3)(x-2)(x-3)$

   e. Roots at -2, 0 and double root at 1

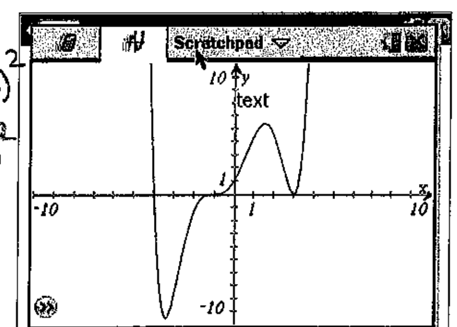
$y = x(x+2)(x-1)^2$

   f. Roots at -2, 0 a double root at 1, a triple root at 2 and goes through the point (3,2).

$y = a \cdot x(x+2)(x-1)^2(x-2)^3$
$y = \frac{1}{30}x(x+2)(x-1)^2(x-2)^3$

   g. For the graph

$y = a \cdot (x+4)(x+1)(x-3)^2$
$y = \frac{1}{36}(x+4)(x+1)(x-3)^2$



[4] MAT150 College Algebra