

### Exploring Vertical Asymptotes & Roots

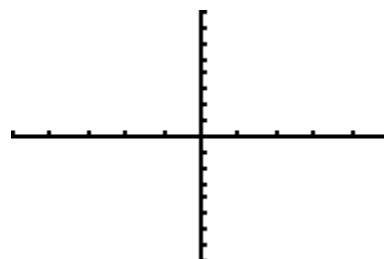
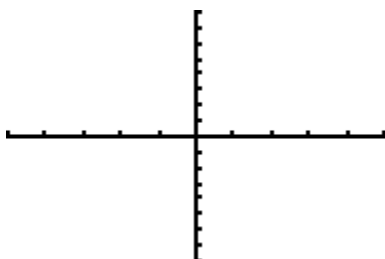
#### Exploration 1:

A. Consider  $f(x) = x^2 + x - 6$  and  $g(x) = x + 2$ . As accurately as possible, graph these functions on the given grids below. The window settings are given.

$$f(x) = x^2 + x - 6$$

$$g(x) = x + 2$$

```
WINDOW
Xmin=-5
Xmax=5
Xscl=1
Ymin=-8
Ymax=8
Yscl=1
↓Xres=■
```



B. What are the roots/zeros of each function? Describe how you found them both graphically and algebraically. *Hint: factor f(x)*

$$f(x) = x^2 + x - 6$$

$$g(x) = x + 2$$

C. Use a graphing calculator to graph  $h(x) = \frac{f(x)}{g(x)}$ .

Be careful to put parentheses around the entire numerator and denominator.

Looking at the table of values...

$$Y1 = ( \quad ) / ( \quad )$$

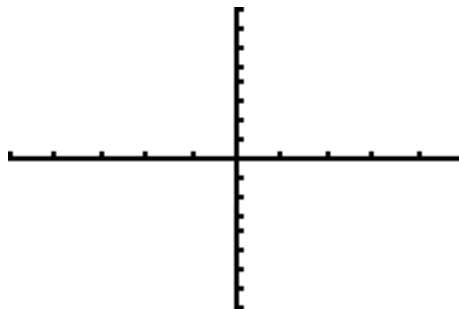
Where are the roots/zeros of  $h(x)$ ?

\_\_\_\_\_

Where is  $h(x)$  undefined?

State the vertical asymptotes.

\_\_\_\_\_



Write  $h(x)$  in factored form:

$$h(x) = \frac{f(x)}{g(x)} = \frac{\quad}{\quad}$$

D. Use a graphing calculator to graph  $p(x) = \frac{g(x)}{f(x)}$ .

Be careful to put parentheses around the entire numerator and denominator.

Looking at the table of values...

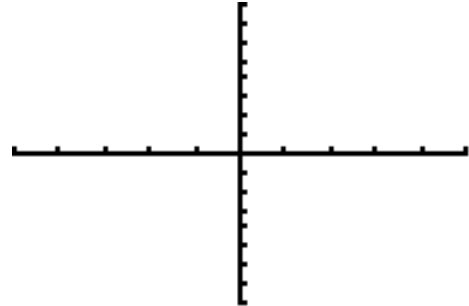
$$Y1 = ( \quad ) / ( \quad )$$

Where are the roots/zeros of  $p(x)$ ?

\_\_\_\_\_

Where is  $p(x)$  undefined?  
State the vertical asymptotes.

\_\_\_\_\_



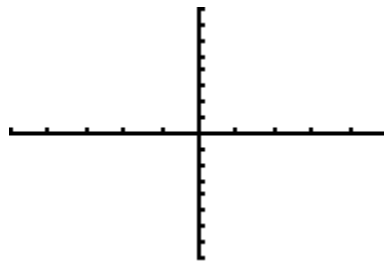
Write  $p(x)$  in factored form:  $p(x) = \frac{g(x)}{f(x)} =$  \_\_\_\_\_

### Exploration 2:

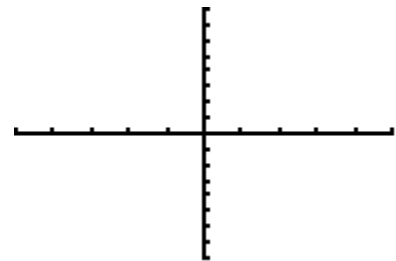
A. Consider  $f(x) = x^2 - x - 2$  and  $g(x) = x^2 + 2x - 3$ . As accurately as possible, graph these functions on the given grids below. The window settings are given.

```
WINDOW
Xmin=-5
Xmax=5
Xscl=1
Ymin=-8
Ymax=8
Yscl=1
↓Xres=■
```

$$f(x) = x^2 - x - 2$$



$$g(x) = x^2 + 2x - 3$$



B. What are the roots/zeros of each function? Describe how you found them both graphically and algebraically. *Hint: factor f(x)*

$$f(x) = x^2 - x - 2$$

$$g(x) = x^2 + 2x - 3$$



**B.** What are the roots/zeros of each function? Describe how you found them both graphically and algebraically. *Hint: factor  $f(x)$*

$$f(x) = (x - 3)(x + 2)(x - 1)$$

$$g(x) = x - 2$$

**C.** Use a graphing calculator to graph  $h(x) = \frac{f(x)}{g(x)}$ .

Be careful to put parentheses around the entire numerator and denominator.

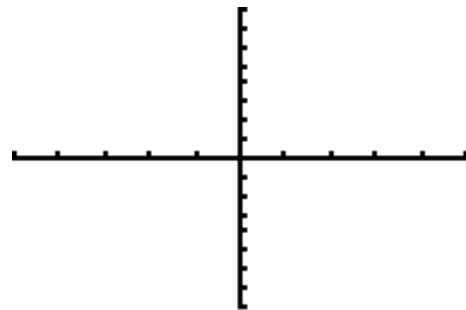
**Looking at the table of values...**  $Y1 = [ \quad \quad \quad ] / ( \quad \quad )$

Where are the roots/zeros of  $h(x)$  ?

\_\_\_\_\_

Where is  $h(x)$  undefined?  
State the vertical asymptotes.

\_\_\_\_\_



Write  $h(x)$  in factored form:

$$h(x) = \frac{f(x)}{g(x)} = \frac{\quad \quad \quad}{\quad \quad \quad}$$

**D.** Use a graphing calculator to graph  $p(x) = \frac{g(x)}{f(x)}$ .

Be careful to put parentheses around the entire numerator and denominator.

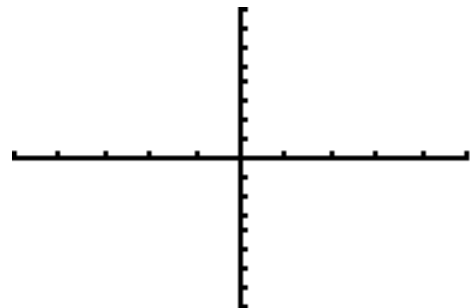
**Looking at the table of values...**  $Y1 = ( \quad \quad ) / [ \quad \quad \quad ]$

Where are the roots/zeros of  $p(x)$  ?

\_\_\_\_\_

Where is  $p(x)$  undefined?  
State the vertical asymptotes.

\_\_\_\_\_



Write  $p(x)$  in factored form:

$$p(x) = \frac{g(x)}{f(x)} = \frac{\quad \quad \quad}{\quad \quad \quad}$$

## **SUMMARY AND CONCLUSIONS about Rational Functions:**

1. The roots of the factors in the numerator correspond with the \_\_\_\_\_ on the graph.
2. The roots of the factors in the denominator correspond with where the function is undefined. This is where \_\_\_\_\_ occur.
3. Given the functions  $f(x) = (x - a)(x + b)(x - c)$  and  $g(x) = (x - d)(x + e)$ 
  - A. Where are the ROOTS of the function  $h(x) = \frac{f(x)}{g(x)}$  ?
  - B. Where are the VERTICAL ASYMPTOTES of  $h(x) = \frac{f(x)}{g(x)}$  ?
  - C. Where are the ROOTS of the function  $p(x) = \frac{g(x)}{f(x)}$  ?
  - D. Where are the VERTICAL ASYMPTOTES of  $p(x) = \frac{g(x)}{f(x)}$  ?
4. What type of functions were  $f(x)$  and  $g(x)$ ?
5. In all of the explorations above,  $h(x)$  and  $p(x)$  are called **rational functions**. Can you formulate a definition for a rational function?
6. For any simplified rational function, what information can you obtain from the numerator?
7. What information can you obtain from the denominator?