Rational Functions Activity #2
Exploring Removable Discontinuities

Quick Check:

If you can correctly answer the questions below, you are ready to move onto Activity #2.

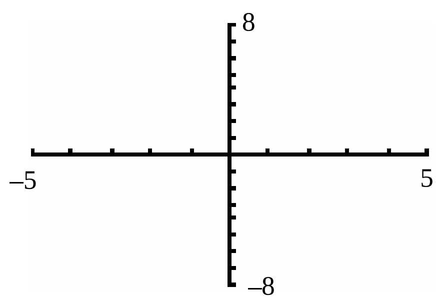
1. Determine which of the following are rational functions. State why you chose your answer.

$$p(x)=\frac{x^{1/2}+2x+1}{x^2+2x-1} \qquad g(x)=\frac{2x^3+x^2-1}{x+3} \qquad h(x)=\frac{2^x+2}{x+2}$$

2. Given $f(x)=\dfrac{6x^2-13x-5}{x^2-5x+4}$, find
    1. The domain of $f(x)$.
    2. The roots of $f(x)$.
    3. The vertical asymptotes, or infinite discontinuities, of $f(x)$.

Exploration #1

a) Use your graphing calculator to graph $f(x)=x^2+x-6$ and $g(x)=x-2$ on the same viewing window. As accurately as possible, sketch the graph of these functions on the given grid below. Where do the graphs intersect?



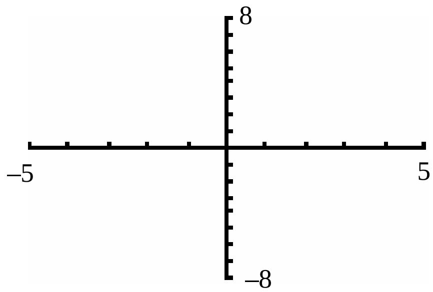
b) Let $h(x)=\dfrac{f(x)}{g(x)}=\dfrac{x^2+x-6}{x-2}$. Rewrite $h(x)$ with its numerator factored.

- What do you notice about the factor $x-2$?
- What type of function results when $h(x)$ is simplified?
- What do you think the graph of $h(x)$ will look like? State specifically what you think the graph will look like at $x = 2$.

c) Use your graphing calculator to graph $h(x)$. Use ZOOM → #4 decimal and then adjust the window settings so ymin = –6.2 and ymax = 6.2.

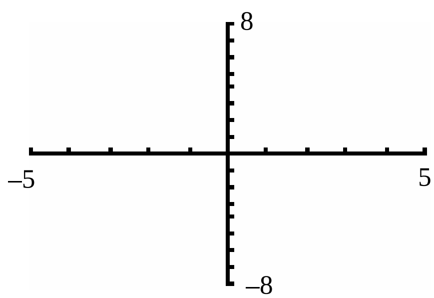
- Does the graph look different at $x = 2$? Describe what you see.
- What does the table show at $x = 2$?

d) Give an accurate sketch of $h(x)$ on the grid below.



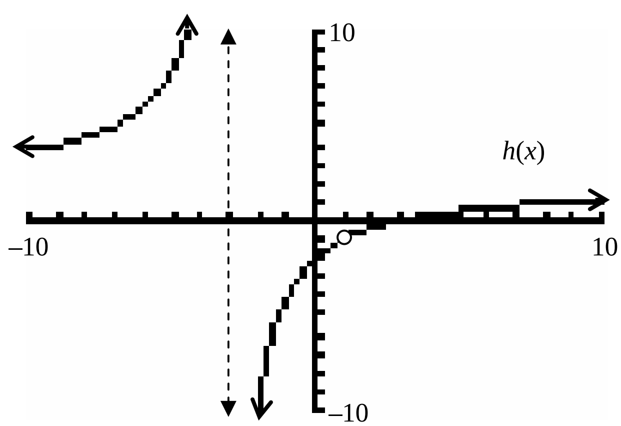
e) Without using your calculator, what do you think the graph of $p(x) = \dfrac{g(x)}{f(x)}$ would look like? List all intercepts, vertical asymptotes (infinite discontinuities), and holes (removable discontinuities).

f) Use your graphing calculator to graph $p(x)$. Use ZOOM → #4 decimal and then adjust the window settings so ymin = –6.2 and ymax = 6.2, and then accurately draw the graph of $p(x)$ on the grid below.



Exploration #2

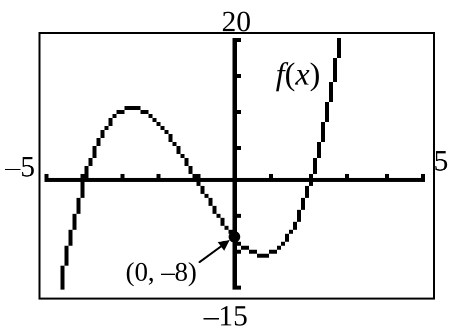
a) A graph of $h(x)$ is given below.

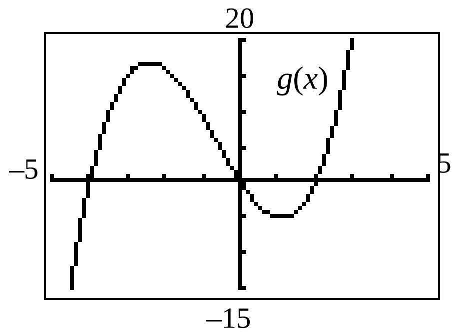


- Give the roots of the function.
- Give the *y*-intercept of the function.
- Give the infinite discontinuities of the function.
- Give the removable discontinuities of the function.
- Give a possible equation for $h(x)$.

Exploration #3

Graphs of $f(x)$ and $g(x)$ are given below.



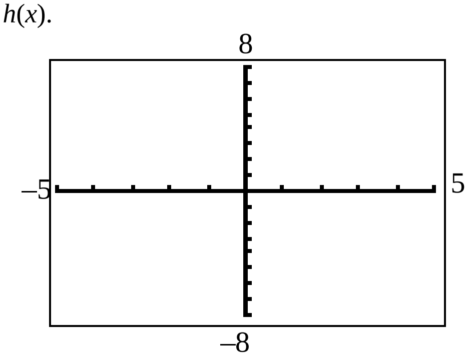


a) What are the roots of $f(x)$?

b) What are the roots of $g(x)$?

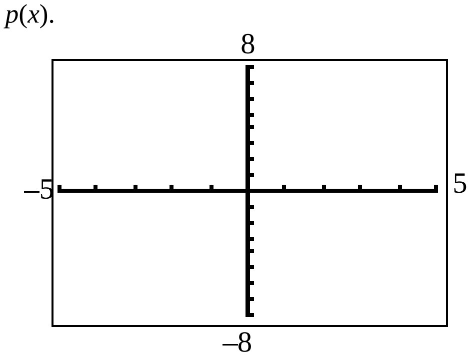
c) Let $h(x) = \dfrac{f(x)}{g(x)}$.

- What are the roots of $h(x)$?
- Does $h(x)$ have any infinite discontinuities (vertical asymptotes)?
- Does $h(x)$ have any removable discontinuities (holes)?
- Without using your calculator, sketch a graph of $h(x)$. (Use the grid below.)
- Give a possible equation for $h(x)$.



d) Let $p(x) = \dfrac{g(x)}{f(x)}$.

- What are the roots of $p(x)$?
- Does $p(x)$ have any infinite discontinuities (vertical asymptotes)?
- Does $p(x)$ have any removable discontinuities (holes)?
- Without using your calculator, sketch a graph of $p(x)$. (Use the grid below.)
- Give a possible equation for $p(x)$.



Summary and Conclusions

1. Given functions $f(x)=(x-a)(x+b)(x-c)$ and $g(x)=(x-a)(x+b)(x-d)$ where a, b, c, and d are positive real numbers such that $a \neq b \neq c \neq d$,

   a) Where are the roots of the function $h(x)=\dfrac{f(x)}{g(x)}$?

   b) Where are the vertical asymptotes (infinite discontinuities) of the function $h(x)=\dfrac{f(x)}{g(x)}$?

   c) Where are the removable discontinuities of $h(x)=\dfrac{f(x)}{g(x)}$?

   d) Where are the roots of the function $p(x)=\dfrac{g(x)}{f(x)}$?

   e) Where are the vertical asymptotes (infinite discontinuities) of the function $p(x)=\dfrac{g(x)}{f(x)}$?

   f) Where are the removable discontinuities of $p(x)=\dfrac{g(x)}{f(x)}$?

2. What do you think is a good procedure to follow when analyzing and graphing ANY rational function? By analyzing, we mean find the domain, roots, and any discontinuities (infinite or removable). Explain why you would formulate this procedure.