Rational Functions Activity #3
Exploring Horizontal Asymptotes

Quick Check:

Exploration #1 – Horizontal Asymptotes

a) Use your graphing calculator to graph $f(x)=\dfrac{3x+1}{x+1}$.

- Go to your table (2^{nd} GRAPH), scroll down and examine the *f*(*x*) values as the *x* values tend toward positive infinity. What do you see? How would you use mathematical notation (symbols) to write this?

- Go to your table, scroll up and examine the *f*(*x*) values as the *x* values tend toward negative infinity. What do you see? How would you use mathematical notation (symbols) to write this?

- Press the TRACE button on your calculator and use the arrow buttons to trace the graph as the *x* values tend toward positive infinity. What is happening to the function's values as you trace along the graph? Is this matching up with what you found above?

- Press the TRACE button on your calculator and use the arrow buttons to trace the graph as the *x* values tend toward negative infinity. What is happening to the function's values as you trace along the graph? Is this matching up with what you found above?

- Look at the equation for *f*(*x*). Can you explain why the function's values get closer and closer to the value found above as the *x* values tends toward positive and negative infinity?

- Do the *f*(*x*) values have this same value for small *x* values (*x* = –5, –4, –3, –2, …..3, 4, 5)? Why or why not?

If the function values get closer and closer to a certain value, *L*, as *x* tends to either positive or negative infinity, we say a *horizontal asymptote* exists at *y* = *L*.

- What is the equation for the horizontal asymptote of *f*(*x*) above?

b) Use your graphing calculator to graph $h(x)=\dfrac{4x^2+1}{-2x^2-3}$.

- Go to your table, scroll down and examine the $h(x)$ values as the x values tend toward positive infinity. What do you see? How would you use mathematical notation (symbols) to write this?

- Go to your table, scroll up and examine the $h(x)$ values as the x values tend toward negative infinity. What do you see? How would you use mathematical notation (symbols) to write this?

- Press the TRACE button on your calculator and use the arrow buttons to trace the graph as the x values tend toward positive infinity. What is happening to the function's values as you trace along the graph? Is this matching up with what you found above?

- Press the TRACE button on your calculator and use the arrow buttons to trace the graph as the x values tend toward negative infinity. What is happening to the function's values as you trace along the graph? Is this matching up with what you found above?

- What is the equation for the horizontal asymptote of $h(x)$ above?

- Look at the equation for $h(x)$, can you explain why the function's values get closer and closer to the value found above as the x values tends toward positive and negative infinity?

- Do the $h(x)$ values have this same value for small x values ($x = -5, -4, -3, -2, \ldots..3, 4, 5$)? Why or why not?

c) Use your graphing calculator to graph $p(x)=\dfrac{2x^3+4x^2-6x}{x^3+x^2-16x+20}=\dfrac{2x(x+3)(x-1)}{(x-2)^2(x+5)}$. Does $p(x)$ appear to have a horizontal asymptote? If so, give the equation of the asymptote?

d) Use your graphing calculator to graph $q(x)=\dfrac{x+2}{2x^2-2}=\dfrac{x+2}{2(x+1)(x-1)}$. Does $q(x)$ appear to have a horizontal asymptote? If so, give the equation of the asymptote?

e) Use your graphing calculator to graph $r(x)=\dfrac{2x^2+5x-3}{x^3+3x^2+2x}$. Does $q(x)$ appear to have a horizontal asymptote? If so, give the equation of the asymptote?

f) Use your graphing calculator to graph $s(x)=\dfrac{2x^3+x^2-1}{x^2}$. Does $q(x)$ appear to have a horizontal asymptote? If so, give the equation of the asymptote?

g) Use your graphing calculator to graph $y(x)=\dfrac{-3x^3+x^2-1}{x^2-1}$. Does $q(x)$ appear to have a horizontal asymptote? If so, give the equation of the asymptote?

h) Record your finding in a-g above in the following table.

Function	Leading Term in Numerator	Leading Term in Denominator	Equation for Horizontal Asymptote
a) $f(x)=\dfrac{3x+1}{x+1}$			
b) $h(x)=\dfrac{4x^2+1}{-2x^2-3}$			
c) $p(x)=\dfrac{2x^3+4x^2-6x}{x^3+x^2-16x+20}=\dfrac{2x(x+3)(x-1)}{(x-2)^2(x+5)}$			
d) $q(x)=\dfrac{x+2}{2x^2-2}$			
e) $r(x)=\dfrac{2x^2+5x-3}{x^3+3x^2+2x}$			
f) $s(x)=\dfrac{2x^3+x^2-1}{x^2}$			
g) $y(x)=\dfrac{-3x^3+x^2-1}{x^2-1}$			

- Some patterns/relationships exist in the table above that will help you determine the horizontal asymptote of a rational function. Can you find them?

- Use the findings of the table and give an equation for of a rational function that satisfies the following.(do not use any of the rational functions given above)
  i) Has a horizontal asymptote at $y = -5$. equation:________________________________

  ii) Has a horizontal asymptote at $y = 0$. equation:________________________________

  iii) Does not have a horizontal asymptote. equation:________________________________

  iv) Has a root at $x = 4$, a hole at $x = -3$, a vertical asymptote at $x = -1$, and a horizontal asymptote at $y = 2$.

  equation:________________________________

Summary and Conclusions

1. What does a horizontal asymptote tell us about a function?

2. Can a function cross a horizontal asymptote (hint: look at the graph of $p(x)$ from above)? Why or why not?

3. What are the differences between a horizontal and a vertical asymptote?

4. How can you find the horizontal asymptotes of a rational function given its graph?

5. How can you find the horizontal asymptotes of a rational function given its table of values?

6. How can you find the horizontal asymptote of a rational function given its equation?